INTERPRETATION OF THE LACTATION CURVE.

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(Accepted for publication, June 9, 1926.)

INTRODUCTION.

The term lactation curve is used to refer to the curve representing the rate of milk secretion with advance in lactation. It has been known for some years that the group lactation curve for the dairy cow is of a descending exponential type. Sturtevant first published herd data from which he concluded that the milk yield for any month showed a decrease of about 9 per cent as compared with the month preceding. Brody, Ragsdale, and Turner, seeking to correlate the course of milk secretion with the laws governing the rate of chemical reactions, have expressed the lactation curve in the form of an exponential equation, \( M_t = M_0 e^{-kt} \), in which \( M_t \) is the rate of yield at any time, \( t \), and \( M_0 \) is the initial rate of yield. This is the type of equation representing the course of a monomolecular reaction in which \( M_0 \) would represent the initial amount of the substance undergoing such reaction and \( M_t \) would represent the amount of the substance remaining unchanged at any time, \( t \). This similarity in form of the equations Brody et al. interpret in favor of a limiting substance governing the maximum rate of milk secretion at any stage of lactation, the limiting substance undergoing monomolecular change which inactivates it so far as its effect on the rate of milk secretion is concerned.

The above results are based on the raw data of milk yield. It has been shown by Gaines and Davidson that the equation applies even more closely to the data of energy yield than it does to milk yield. While the equation is admirably adapted to describe the lactation

curve in simple mathematical terms, the chemical interpretation is open to question. From the viewpoint of Brody et al. the factor \( k \) of the equation corresponds to the specific velocity constant of the chemical reaction. It is the purpose of the present paper to present some results showing the \( k \) values of individual lactation curves as bearing on the interpretation to be placed on the group behavior.\(^4\)

\[ \frac{dy}{dt} = Ae^{-kt} \]

Fig. 1. Frequency distribution of 1534 Guernsey records with respect to rate of decrease per month \( (k) \) in rate of yield.

**Individual Lactation Curves.**

A curve of the type \( \frac{dy}{dt} = Ae^{-kt} \) has been fitted by an adaptation of the method previously described to each of 1676 Guernsey records. In the equation \( y = \) yield and \( t = \) time in months. Yield for a month

\(^4\)The data here given are taken from a manuscript presenting a broader investigation of the \( k \) values, which has been submitted for publication to the Agricultural Experiment Station.
represents the rate of yield at the middle of that month, with an entirely negligible error. Energy value of the milk solids had been used as being the most fundamental of the several available measures of yield. The distribution of the $k$ values of these records, excluding 142 which were highly irregular, are given in Fig. 1. For a chemical interpretation of the lactation curve we may consider that the data of Fig. 1 represent 1534 determinations of the velocity constant $k$.

It will be noted from Fig. 1 that the $k$ values fall into a quite regular order approaching a normal distribution. The mean of the array is $0.04425 \pm 0.00055$; standard deviation, $0.03219 \pm 0.00089$; and coefficient of variability, 72.75 $\pm$ .89. There is thus shown a high degree of variability in the $k$ constants.

**Ascending Lactation Curves.**

It is to be noted further that 83 of the lactation curves, 5.41 per cent of the total, show negative $k$ values, that is, the slope is positive. The velocity constant is not only highly variable, but apparently even reverses its sign in an appreciable number of cases. Obviously some modification of the simple monomolecular interpretation is necessary.

The group behavior of these ascending records is not in good conformity with the equation type, being somewhat aberrant at the start and finish. Indeed, to anyone conversant with the great energy transformations performed by the lactating cow it would be absurd to suppose that the lactation curve could continue to ascend for more than a limited time. The point that any hypothesis must satisfy is that within this group the curve does ascend for 9 or 10 months.

**Irregular Lactation Curves.**

Under this head are to be considered the 142 records not included in Fig. 1 and necessary to complete a representative sample of the Guernsey records. The average of this group of irregular records is given in Fig. 2 together with 4 individual curves to show the diversity of the records that enter into the average. If the monthly yields are plotted on a logarithmic scale against time on an arithmetic scale,
they should fall on a straight line to satisfy the equation. The data are plotted in Fig. 2 to show the degree of conformity to this relation.

The 142 records which we are now considering are so irregular that they were rejected from the main study on the ground that they must have been unduly influenced by extraneous factors. The point of interest in the present connection brought out in Fig. 2 is, that treated

as a group we get a result conforming acceptably with the equation. The meaning of the regularity of the group behavior is not clear, when the group is composed of such variable individual elements. It means that the individual deviations from the type curve are of a compensating nature. On the theory of a limiting substance determining the maximum rate of milk secretion, it is easy to account for minus deviations through the effect of other factors in preventing
realization of the maximum, but the difficulty lies in accounting for the plus deviations which serve to counterbalance the minus deviations.

Factors Affecting the Rate of Decrease.

That the $k$ values of the lactation curve equations are greatly affected by conditions of feeding and management of the herd is sufficiently evident from the difference in the performance under advanced registry conditions and conditions of commercial milk production. It may be presumed that the same cows that show under advanced registry a mean $k$ value of .044 would show under the less favorable commercial conditions a mean value of .09 to .1. Obviously nutritional conditions are a powerful factor affecting the rate of decrease.

The $k$ values are also closely related to the initial rate of yield, the correlation between $k$ and $A$ being $r = .535 \pm .012$. On the limiting substance theory the initial quantity of this substance is directly proportional to $A$. The limiting substance is assumed to disappear in accordance with a monomolecular reaction. The velocity constant of the reaction must be assumed, therefore, to vary directly with the initial amount of the limiting substance, in order to satisfy the observed results. One of the laws of unimolecular reaction is that the velocity constant is the same regardless of the concentration of the reacting substance. Consequently, the observed relation between $k$ and $A$ is in conflict with the theory of a limiting substance and its monomolecular inactivation.

On the other hand, regarded as a nutritional matter and bearing in mind the large energy requirements of lactation, it is not surprising that the rate of decrease should be greater the greater the initial rate of yield.

SUMMARY.

The validity of the assumption of a substance determining the rate of milk secretion and undergoing monomolecular destruction, based on group behavior, is questioned on the evidence from a large number of individual lactation curves. It seems probable that the rate of decrease in the rate of milk secretion with advance in lactation is dependent upon factors of a nutritional nature.