Conjoint Control of Hippocampal Place Cell Firing by Two Visual Stimuli II. A Vector-field Theory that Predicts Modifications of the Representation of the Environment

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abstract Changing the angular separation between two visual stimuli attached to the wall of a recording cylinder causes the firing fields of place cells to move relative to each other, as though the representation of the floor undergoes a topological distortion. The displacement of the firing field center of each cell is a vector whose length is equal to the linear displacement and whose angle indicates the direction that the field center moves in the environment. Based on the observation that neighboring fields move in similar ways, whereas widely separated fields tend to move relative to each other, we develop an empirical vector-field model that accounts for the stated effects of changing the card separation. We then go on to show that the same vector-field equation predicts additional aspects of the experimental results. In one example, we demonstrate that place cell firing fields undergo distortions of shape after the card separation is changed, as though different parts of the same field are affected by the stimulus constellation in the same fashion as fields at different locations. We conclude that the vector-field formalism reflects the organization of the place-cell representation of the environment for the current case, and through suitable modification may be very useful for describing motions of firing patterns induced by a wide variety of stimulus manipulations.

key words: cognitive maps • stimulus control • place fields

INTRODUCTION

The purpose of this paper is to present a quantitative description of how the firing fields of hippocampal place cells are affected by two kinds of stimulus manipulations (Fenton et al., 2000). In one kind ("reconfigurations"), we found that changing the angular distance between a black card and a white card on the wall of a cylinder caused fields to move relative to each other in a systematic fashion, as though the representation of the cylinder floor underwent a topological distortion. These changes in the relative positions of firing fields were accompanied by a position-independent decrease in place cell firing rates, regardless of whether the cards were closer or further apart by 25°. In the second kind of manipulation ("removals"), we saw that deleting either card left the representation of the cylinder floor intact; fields did not move relative to each other nor were their firing rates altered. Both reconfigurations and removals were done after superimposing a 45° rotation of the manipulated cue configuration. In every case, firing fields followed this additional 45° rotation, indicating that stimulus control resided in the two cards and not in background stimuli that are fixed in the environment.

A full account of these results would be based on a neural network model and would address a variety of questions, including the origins of the firing fields in the standard conditions, the lack of effect of card removals on both the relative positions of fields and firing rates, and the ability of card reconfigurations to induce both relative field position movements and position-independent decreases of firing rate. The geometric theory we present here is less ambitious; it is concerned only with field movements and explains neither why fields exist in the first place nor why reconfigurations cause changes in firing rates.

This theory consists of an empirical vector-field equation that relates the movement of all field centroids to weighted functions of the movements of both cards. Field movement is therefore a smooth function of the initial position of the centroid in the cylinder. Thus, neighboring fields are constrained to move in concert and no allowance is made for individual fields to be coupled to arbitrarily selected combinations of the available stimuli. The model, like our data, is therefore in contrast to the virtually complete independence of fields from each other that was used to describe the effects of counter-rotating distal and proximal cues (Shapiro et al., 1997; Tanila et al., 1997; see also Brown and Skaggs, 1999).

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Beyond providing a concise summary of field movements, the vector-field theory is valuable in several ways. First, it serves as a benchmark against which the field-movement predictions of any network theory can be checked. Second, we will show that the vector-field equation predicts additional features of our experimental data that were not included in the equation. In our view, the success of these predictions indicates that the stimulus cards act in a smooth, continuous way on the positional activity distributions of hippocampal neurons located everywhere in the environment and that the undistorted fields in the standard configuration therefore serve to indicate location in space rather than the conjunction of stimuli important for each cell.

**METHODS**

Our goal is to write an empirical vector-field equation that describes how the center of a firing field anywhere in a cylindrical apparatus moves when a white stimulus card and a black stimulus card on the cylinder wall are both moved or one is deleted and the other is moved. Thus, we want to calculate \( V \), the displacement vector for the field center (Eq. 1):

\[
V = f(X, V_b, V_w),
\]

where \( X \) is the initial location of the field and \( V_b \) and \( V_w \) are vectors that point from the initial to the altered locations of the black and white cards (see Fig. 1 for notation).

We first treat rotations of field centers induced by moving a card. During rotations of a single card (Muller and Kubie, 1987) or equal rotations of both cards, fields act as though they are rigidly connected to a pivot at the apparatus center and the pivot is turned through the same angle by which the stimuli are rotated. For instance, the rotational field movement that would be induced by the black card is \( V_b \). The length of this vector is given by Eq. 2:

\[
|V_b| = (r/R)|V|,
\]

where \( R \) is the radius of the cylinder and \( r \) is the distance of the field from the cylinder center; the direction of the vector is such that the angular movement of the field is equal to the angular movement of the black card. Similarly, the rotational field movement that would be induced by the white card is \( V_w \).

If the two cards always contributed equally to rotations of field centers, the net rotational movement of the field center would be the average of the vectors \( V_b \) and \( V_w \). We saw from reconfiguration experiments, however, that the contribution of the card near a field was greater than the contribution of the other card. We imagine, therefore, that the contribution of a card is inversely proportional to the distance of the field from that card. The net rotational movement, \( V_n \), is the weighted average of the rotational field movements that would be separately induced by each card and is given by:

\[
V_n = \left( \frac{d_w V_b + d_b V_w}{d_w + d_b} \right) / (d_w + d_b),
\]

where \( d_w \) is the distance from the field center to the white card and \( d_b \) is the distance from the field center to the black card as show in Fig. 1. Eq. 3 correctly predicts that equal rotations of the two cards causes equal rotations of all field centers. Under the interpretation that deleting a card makes the distance from the card to any field arbitrarily large, Eq. 3 also correctly predicts that after removing one card all fields rotate equally with the remaining card. Moreover, it works well for fields near either card. Interestingly, Eq. 3 describes how field centroids would move in the case that each card controls an independent component of the field and is considered briefly in the results.

Although Eq. 3 performs acceptably near the cards, it incorrectly predicts the movements of fields between the two cards. Thus, regardless of whether the cards are moved apart or together, Eq. 3 predicts that the net movement of such fields is toward the cylinder center rather than nearly parallel to the motion of the line that connects the card centers. To compensate for this error, we define a translational vector for fields, \( V_T \), given by:

\[
V_T = (V_b + V_w)/\left( c_1 d_w d_b + c_2 (1/d_w + 1/d_b) \right).
\]

The direction of \( V_T \) is always parallel to the direction of the motion of the line that connects the two card centers. The two terms in the denominator of Eq. 4 serve different purposes. The first makes \( V_T \) zero if \( d_w \) or \( d_b \) becomes arbitrarily large when the corresponding card is removed. The constant \( c_1 \) is very small so that this term has no effect except during card removal. The second term in the denominator reduces the effect of the translational term as the distance to either card decreases, thereby preserving the effects of the rotational term. The constant \( c_2 \) allows the amount of translation caused by card movements to be adjusted independently of the amount of rotation caused by card movements. The total motion of the fields, \( V \), is the sum of \( V_n \) and \( V_T \):

\[
V = V_n + V_T = (d_w V_b + d_b V_w)/(d_w + d_b) + (V_b + V_w)/(c_1 d_w d_b + c_2 (1/d_w + 1/d_b)).
\]

The patterns of field movements generated by Eq. 5 for apart and together card manipulations are shown in Fig. 2 in the normalized coordinate system (Fenton et al., 2000). In the same coordinate system, the corresponding pictures for equal card rotations and card removals would be just sets of zero-length vectors, indicating no systematic field motion. By inspection of Figures 2, 6A, and 7A of Fenton et al. (2000), predictions of the vector-field equation resemble the empirical patterns of field movements after reconfigurations. We therefore turn to a numerical analysis.

**RESULTS**

Predictions of a Simple Component Model for Field Centroid Movements

Changes in the size or aspect ratio of a recording box can induce changes in the size and shape of firing fields (Muller and Kubie, 1987; O’Keefe and Burgess, 1996). In some cases, lengthening the box can cause a field with a single peak to develop two peaks or even to come apart into two pieces as though the field had two components, one associated with a wall and the other with the opposite wall (O’Keefe and Burgess, 1996). When the walls are separated by their standard distance, the two components superimpose, but their existence is revealed when the wall separation is changed. Similar effects were seen for some fields in the double rotation experiments of Shapiro et al. (1997).

Might card reconfigurations also split fields into two components? We saw no such effect in firing rate maps,
clusively by the white (black) card. Note that the resultant of the centroid that would occur if the field position were controlled ex-
center regardless of whether the cards are moved apart or to-
25° Apart

B1

25° Together

Figure 1. Diagrammatic definitions of quantities in Eq. 5. (A) The open and filled circles on the circumference of the cylinder represent, respectively, the centers of white and black cards in their standard positions. X is the position of a field centroid. d_w and d_b are, respectively, the distance from the field centroid to the centers of the white and black cards. B, 1 (2), shows the positions of the card centers after moving the cards 25° apart (25° together). The vector V_w (V_b) shows the purely rotational movement of the field centroid that would occur if the field position were controlled exclusively by the white (black) card. Note that the resultant of the vectors V_w and V_b is small and points inward towards the cylinder center regardless of whether the cards are moved apart or together. Thus, any model that predicts the field movement to be the vector sum of the rotations of the stimulus movements is incorrect.

but the simplicity of the stimulus arrangement makes it easy to test the component hypothesis directly. We assume that both putative components are in register when the cards are in their standard positions. We also assume, from card removals, that each component undergoes a pure rotation caused by rotation of the relevant card. By symmetry, the centroid of the composite field must be halfway between the centroids of the two components, so the composite centroid can experience only radial, but not angular movements. In addition, this radial movement can only be inward, regardless of the angular position of the field, an effect clearly at odds with our data.

In this simple component model, the weakening of control with distance between the card and the field is not included. We have, however, already implicitly con-
sidered a component model that includes the distance effect, expressed by Eq. 3. Solutions to Eq. 3 are shown as gray lines in Fig. 3, where it is seen that fits to the an-
gular centroid movements are not very bad, but fits to the radial movements are unacceptable. It was the failure of Eq. 3 that led us to add the translational term.

From this analysis, we conclude reconfigurations do not split fields into components that are controlled by the two cue cards. It is important to note that this analysis militates against any model in which the two cards separately trigger activity in the place cells. Thus, we argue that schemes in which control over firing resides at some times with one card and at other times with the second card are not accurate. Schemes of this sort include those in which the animal resets the coordinate system relative to one card, and then uses only self-motion information until another reset relative to the other card.

Fit of the Vector Field Equation to Field Centroid Movements

The ability of Eq. 5 to reproduce field centroid movements after the cue cards are moved apart is shown by the black lines in Fig. 3 A, 1 and 2. The observed angular (Fig. 3 A, 1) and radial (2) displacement vector components are the same as in Figure 6 B, 1 and 2, of Fenton et al. (2000), but the sine waves used to organize the data in the previous paper have been replaced by solutions to Eq. 5 (black lines). These solutions are for c_2 = 83.2 and for a constant value of the radial position; namely, the circle that divides the cylinder into a central disk and outer annulus of equal area. Solutions at other radial positions (not shown) vary in shape, but each substantially reproduces the pattern of data.

A correlational analysis indicates a satisfactory fit of Eq. 5 to the angular and radial displacements caused by apart card movements. We first calculated the correlation between two measures of the angular movement of field centers in apart sessions; namely, the observed movement compared with standard sessions and the movement expected from the vector-field equation given field center positions in the previous standard session. According to a t test, the probability that the correlation of 0.774 with 45 df occurred by chance was 1.78 \times 10^{-10}. The correlation of 0.721 with 45 df between observed and predicted radial movements associated has a probability of 1.08 \times 10^{-8}. Thus, the theory accounts for 60% of the variance in angular field movement and 52% of the variance in radial field movement caused by moving the cards apart without any correction for random movements of field centers between pairs of standard sessions (Fenton et al., 2000).

The ability of Eq. 5 to account for field centroid movements when the cards are moved together is shown by the black lines in Fig. 3 B, 1 and 2. Once again, the solutions are for c_2 = 83.2 and for the circle that divides the cylinder into a central disk and an annulus of equal area. A correlational analysis confirms that the fit is very good for the angular component of
Solutions of the Vector Field Equation
(Equation 5)

Figure 2. The arrows represent solutions of the vector-field equation associated with 25° apart and 25° together movements of the cue cards. The lengths and directions of the arrows vary with the initial position of the field centroid in the cylinder. Variations in length can be visualized by the noncollinear positions of arrowheads that arise from vertically aligned vector tails.

Figure 3. Solutions of Eqs. 5 (black line) and 3 (gray line) superimposed on the angular (A, 1 and B, 1) and radial components (A, 2 and B, 2) of observed displacement vectors for apart (A) and together (B) card manipulations. The fit of the Eq. 5 to the radial component of the displacement vectors caused by together card movements could be improved by changing $c_3$ in Eq. 5.
served horizontal displacement, the corresponding 
ment for each observed horizontal displacement, the corresponding 
ments for together sessions was greater than the magnitude 
theoretical vectors. (Inset) Definitions of the vertical and horizontal directions for field movements. The solid 
the line between the two cards moves a somewhat greater distance 
the line that connects the centers of the cue cards.

Differences of Field Movements Induced by Moving the Cards Apart and Together

As seen in Fig. 2, the vector-field equation makes two specific predictions concerning the movements of fields after apart and together card manipulations. First, movements parallel to the horizontal diameter (Fig. 4) should be in opposite directions and, second, there should be no average movement in the vertical direction. In addition, numerical solutions of Eq. 5 indicate that the magnitude of the movement in the horizontal direction should be slightly greater for apart sessions than together sessions. The source of this difference is in the translational term of Eq. 5. The mean horizontal displacement from the translational term for the initial field positions of the theoretical vectors in Fig. 2 is $-4.16$ cm for apart sessions and $+3.80$ cm for together sessions. Since the mean contribution of the rotational term of Eq. 5 is equal in magnitude ($1.10$ cm) for apart and together sessions and of the same sign as for the translational term, the total expected horizontal displacement is $-5.95$ cm for apart sessions and $5.59$ cm for together sessions.

The observed horizontal and vertical centroid displacements for apart and together sessions are summarized in Fig. 4. In agreement with theory, the average vertical displacements of field centroids for apart ($0.147$ cm) and together ($0.387$ cm) sessions were not reliably different from each other ($t = 0.38; df = 109; P(t \geq 0.38) = 0.71$), nor was either reliably different from zero. Also, as expected, the mean horizontal centroid displacement was negative for apart sessions ($-4.84$ cm) and positive for together sessions ($+1.80$ cm), and the magnitude of the horizontal displacement for apart sessions was greater than the magnitude of the horizontal displacement for together sessions ($t = 4.82; df = 109; P(t \geq 4.82) = 4.6 \times 10^{-6}$). Thus, the difference in horizontal displacement is in the expected direction, but is much larger than predicted by theory. The origin of this discrepancy is the small average horizontal displacement caused by together card movements. We showed this by determining, for each observed horizontal displacement, the corresponding predicted displacement. We then did paired t tests between observed and expected horizontal displacements for apart and together sessions. For apart sessions, the average difference between observation and expectation was 0.55, so that the observations were somewhat smaller than expected. This discrepancy was not, however, statistically reliable ($paired t = 1.14; df = 46; P(t \geq 1.14) = 0.26$). In contrast, the average difference between the observed and expected horizontal displacements for together sessions was $-3.77$ cm, so that the observed displacements were considerably smaller than expected ($paired t = 5.67; df = 63; P(t \geq 5.67) = 3.6 \times 10^{-9}$). Similar paired t calculations for the vertical di-
placements show that the average difference between observation and expectation is not reliably different for either apart or together card movements.

In summary, the directions and magnitudes of the horizontal and vertical field centroid displacements caused by apart and together card manipulations are in good agreement with theory. The only discrepancy is the smaller than expected average horizontal movement seen after together manipulations, an effect we are currently unable to explain. One way of handling this discrepancy would be to separately analyze apart and together sessions, allowing us to choose a larger value of the constant $c_2$ in Eq. 5 for together sessions. In the absence of a reason to expect different horizontal movements and for parsimony, however, we used only a single form of Eq. 5. In this regard, we note that the accuracy of additional predictions from the vector-field equation (see below) might have been improved by treating apart and together manipulations separately. It turns out, however, that we can make accurate predictions of changes in firing field shapes and positional firing patterns of hippocampal theta cells (interneurons) without complicating the theory. We believe that these predictions are not sensitive to the relatively small field displacements caused by together manipulations. This insensitivity arises because the additional analyses are made after field centroid displacements are subtracted by superimposing the centroid in a manipulated session onto the centroid for a standard session.

Changes of Field Shapes Caused by Card Reconfigurations

Up to now, we have considered only movements of firing field centroids caused by changing the angular distance between the cards. A key finding is that the direction and magnitude of these centroid movements depend on the initial centroid location, so that the representation of the environment seems to be topologically distorted. Firing fields are not points, but occupy significant fractions of the apparatus surface. Moreover, the linear dimensions of firing fields are substantial fractions of the diameter of the cylinder. Imagine, for example, a circular field whose diameter is, say, one third the diameter of the cylinder. Imagine also two other fields whose centroids happen to lie on the opposite ends of the diameter of the circular field. In general, the distance between the centroids of the two other fields would change after the cards were reconfigured.

Does this “tidal” effect apply only to field centroids, or does it operate on an entire field to distort its shape? In other words, do card reconfigurations cause the field to move as a rigid object or does the vector field operate in a smooth, continuous way to stretch the firing field in a predictable fashion? Observing this sort of deformation would be a powerful indication that the vector-field description is valid.

To test whether card reconfigurations deform the positional firing in a way predicted by the vector-field equation, we calculated in two ways the similarity (defined below) of fields recorded in a standard session and in a card reconfiguration session. In the first method, we calculated the similarity when the field in the standard session was moved relative to the reconfigured field as a rigid object, so that neither its shape nor its firing rate contours were altered. In the second method, the field in the standard session was moved according to the vector-field equation, distorting its positional firing pattern. Based on the ability of the vector-field equation to account for the movements of field centroids, we expected that applying the equation to each point in the field would yield a higher similarity.

In line with earlier work, we define similarity as the $z$ transform of the pixel-by-pixel correlation coefficient ($r$) for a pair of positional firing rate patterns (Bostock et al., 1991). To begin, one rate pattern is superimposed on the other and a paired list is made of the time-averaged firing rates in a pixel; a pixel is included in the list only if its rate was greater than zero in at least one session. The initial superimposition is made by moving the centroid of the standard field onto the centroid of the reconfigured field. The similarity is computed, and then recomputed, as the standard field is moved in a square pattern over a nine by nine pixel region centered on the initial position. The similarity of the pair of positional firing patterns is taken as the maximum of all 81 values.

The statistical analysis of similarity values was done by combining apart and together sessions in the following way. Cells recorded in only an apart (13) or a together (32) session were included. For cells recorded in both apart and together sessions (27), a random choice was made to select either type. In the end, the sample consisted of 29 apart and 43 together sessions. Apart and together sessions were combined because similarity averages were nearly equal for both session types.

When the firing field in the standard was rigidly superimposed on the field in the reconfigured session, the average similarity was $1.14$ ($r = 0.781; r^2 = 0.61$). In contrast, when the vector transformation was first applied to the field in the standard session, the average similarity increased to $1.30$ ($r = 0.844; r^2 = 0.71$); this analysis is summarized in the bar graphs of Fig. 5. A paired $t$ value for the rigid and vector transform similarities showed that the vector values were reliably higher ($t = 5.79; df = 71; P(t > 5.79) = 2.1 \times 10^{-7}$).

The significantly higher similarity after vector transformation indicates that Eq. 5 mimics the observed effects of card reconfigurations, but does not indicate the accuracy of the prediction. To address this question, we calculated for each cell the similarity between a pair of
with experimental error, the vector-field equation as two standard sessions resemble each other. Thus, the firing patterns and that those distortions are accurately reproduced by the vector-field equation. To determine how well the vector-field transformation of the standard session reproduced the reconfigured session, the similarity between standard session pairs after rigid translation was also calculated. The mean similarity for pairs of standard sessions of 1.31 (SEM 0.0395) when the field from the standard session was first transformed according to the vector-field equation. The mean similarity for pairs of vector-field transformations of standard and reconfigured sessions, implying that reconfiguration causes distortions of firing patterns and that those distortions are accurately reproduced by the vector-field equation.

Figure 5. Demonstration that distortions of firing field shapes are predicted by Eq. 5. The gray bars show the mean similarity score for standard and reconfigured session pairs derived in two ways; the error bars show SEMs. In the first, the field from the standard session was rigidly superimposed on the field as explained in the text. The mean similarity of 1.14 (SEM 0.043) was reliably lower than the mean similarity of 1.30 (SEM 0.0395) when the field from the standard session was first transformed according to the vector-field equation. The mean similarity for pairs of standard sessions of 1.31 (SEM 0.380) was no higher than for pairs of vector-field transformations of standard and reconfigured sessions, implying that reconfiguration causes distortions of firing patterns and that those distortions are accurately reproduced by the vector-field equation.

standard sessions. Since the similarity of firing fields in identical conditions is limited by accuracy of tracking, discrimination of action potentials and nonideal positional firing (Fenton and Muller, 1998) using pairs of standard sessions establishes a reasonable upper bound for similarity. Thus, if the vector-field transform does only a fair job of accounting for the distortion induced by reconfiguration, the previously computed value of 1.30 should be lower than the similarity for pairs of standard sessions. We find, however, that the mean similarity for pairs of standard sessions of 1.31 (r = 0.843; r² = 0.71) is almost exactly equal to the vector-field transformed similarity (paired t = 0.11; df = 71; P(t ≥ 0.11) = 0.91) (see Fig. 5). Thus, applying the vector-field transform to a standard session produces a distorted pattern that resembles the reconfigured pattern as closely as two standard sessions resemble each other. Thus, within experimental error, the vector-field equation does a virtually perfect job in accounting for the effects of moving the cards closer together or further apart.

Card Reconfigurations Induce Predictable Changes in the Firing Patterns of Hippocampal Interneurons

In addition to pyramidal cells, recordings are often made from interneurons encountered in the CA1 region of the hippocampus. In our experience, these cells are usually found in stratum oriens or superficial stratum pyramidale. On the assumption that we detect cell bodies but not dendrites, these cells are likely to be basket cells (Andersen et al., 1964) or axo-axonic chandelier cells (Freund and Buzsaki, 1996). The properties of these putative interneurons differ from those of pyramidal cells in several ways. Theta cell waveforms are briefer; the wide-band filtered spike duration of theta cells is less than ~350 μs, but greater than ~350 μs for pyramidal cells. Second, the theta cells do not fire the complex spikes (high frequency action potential bursts with decreasing amplitude) that are characteristic of pyramidal cells. Third, the position-independent average firing rate of theta cells is usually >10 spikes/s, much higher than the 1.0 spikes/s for place cells (Ranck, 1973; Fox and Ranck, 1975; Markus et al. 1994). Theta cell firing is also strongly modulated by the state of the hippocampal electroencephalogram (EEG); it generally doubles when the EEG switches from large, irregular activity to theta.

In addition to differences in electrophysiological and temporal firing properties, theta cells show different positional firing properties than pyramidal cells. Most strikingly, since theta cell discharge never shows the long silent intervals characteristic of place cells, theta cells discharge everywhere in the available space. Nevertheless, there are clear, cell-specific variations in positional firing patterns, even for simultaneously recorded theta cells; the firing in higher rate regions is usually 2–2.5× higher than in low-rate regions. On this basis, and because the sizes and shapes of high rate regions resemble place cell firing fields, Kubie et al. (1990) inferred that the positional firing patterns of theta cells in part reflect direct, powerful connections from place cells.

With this background, we asked whether the effects of card reconfigurations could be detected in the positional firing patterns of theta cells. By inspection, these firing patterns were distorted in the fashions expected from the movements of firing field centroids; two examples are given in Fig. 6. We therefore calculated the similarity of a standard session and a reconfigured session for each cell in two ways, by rigidly shifting the pattern in the standard session and by applying the vector-field transformation before shifting. In either case, the standard session pattern was moved relative to the reconfigured pattern and the correlation between the patterns calculated at each step. As before, the similarity was the z
transform of the maximum correlation. We calculated both similarities for 13 theta cells, six recorded during apart sessions and seven during together sessions.

The average similarity after rigid movement of the firing pattern in the standard session onto the reconfigured pattern was 0.583. This is reliably lower than the average similarity of 0.643 seen after applying the vector-field transform to the pattern in the standard session (paired $t = 3.13; df = 12; P(t > 3.13) = 0.0087$), but not significantly different from the similarities based on the vector transformation (paired $t = 1.32; df = 12; P(t > 1.32) = 0.21$). Despite the lack of statistical significance, however, we think that the accuracy of the vector-field transform is lower for theta cells than place cells. The somewhat poorer predictions may be due to the relatively strong dependence of theta cell activity on behavior as well as on position (Kubie et al., 1990).

In addition to combining the apart and together sessions for theta cells, they can be treated separately to see whether the average direction and magnitude of pattern movements are similar to the movements of firing field centroids. To this end, we used the shift of position necessary to maximize the similarity to estimate firing pattern movements. For apart sessions, the mean horizontal shift (Fig. 1 A) was $-4.62$ cm, in excellent agreement with the average horizontal movement of $-4.84$ cm for centroids of place-cell fields during apart sessions. During together sessions, the average horizontal movement of theta cell firing patterns moved $1.49$ cm, again in excellent agreement with the corresponding value for place cells of $1.84$ cm. Thus, the effects of apart and together sessions on theta cells show strong parallels to the effects on place cells. An interesting exception is the lack of change of theta cell firing rate after reconfigurations despite the decrease of place-cell activity.

**DISCUSSION**

We showed that a single vector-field equation accounts for the movements of firing field centers caused by three kinds of stimulus manipulations; namely, rigid rotations of both cards, card removals, and card reconfigurations. We further showed that the vector-field equation predicts three additional effects of card reconfigurations. (a) Field movements parallel to the motion of the line that joins the cards should be greater for apart than together reconfigurations. (b) If the vector-field affects entire firing fields and not just the center, firing fields should stretch according to the same rule that describes motions of field centroids. This prediction was confirmed by showing that superimposing a standard field onto a reconfigured field was more accurate when done with the vector-field equation than when done with rigid translation. (c) Assuming that theta cells (hippocampal interneurons in stratum oriens and stratum radiatum) receive convergent location-specific input from place cells (Kubie et al., 1990), the positional
firing patterns of theta cells should be distorted by card reconfigurations. This effect was observed and quantitatively predicted by the vector-field equation.

Why should a formulation as minimal as the vector-field model perform so nicely? In our view, it reflects the nature of the hippocampal representation of our simplified environment. There are really two issues concerning the nature of the representation. First, it may be true that hippocampal pyramidal cells can encode nonspatial aspects of the environment under more complex circumstances, but, in the pellet-chasing task, the hippocampal pyramidal cells act in many ways as nearly ideal place cells, at least in the spatial domain (Muller, 1996; Eichenbaum et al., 1999; and see Fenton and Muller, 1998, for a discussion of imperfections of place-cell firing in the temporal domain). Thus, in the pellet-chasing task, the firing rate of each place cell decreases with distance from the field centroid and does not vary with head orientation (Muller et al., 1994). These properties of place cells in the pellet-chasing task make it natural to consider a vector-field approach. We note, however, that the existence of more a complicated representation in which pyramidal cell firing reflects behavior as well as position does not preclude a viable vector-field model; all that is necessary is for the positional distribution of behavior to change in a way similar to changes of ideally location-specific activity.

The second aspect of the representation in line with the vector-field approach is more controversial: we think that the mathematical language of the vector-field equation, with its assumption of local smoothness and its inclusion of both stimuli as controllers for fields everywhere in the environment, reflects the underlying nature of the representation. In short, we think the data and model imply that the place-cell representation is truly map-like, so that place-cell discharge takes place in a framework that incorporates features of two- (and possibly three-) dimensional space.

This view differs from the combinatorial (or relational) theory proposed by Eichenbaum and colleagues (see, for example, Eichenbaum, 1996; Eichenbaum et al., 1999) in that place cells are not triggered by an arbitrary selection of stimuli. Our theory differs from the combinatorial model in an even more fundamental way: we think that neighboring firing fields cannot move independent of each other after gentle stimulus manipulations that induce neither partial nor complete remappings. It is a key prediction of our model that firing field movements must appear to be rigid in a small enough region of the apparatus. Thus, within experimental error, we never expect to see fields with coincident centers move apart from each other to follow differential movements of two stimuli. Specifically, our model predicts that the average movement of two such fields will be zero across multiple standard and reconfigured session pairs.

Another difference between our model and the combinatorial model was alluded to in the preceding paper (Fenton et al., 2000); the angular and radial components of field centroid displacements have a spatial organization; they vary systematically with the initial angular position of the field centroid in the cylinder. The observed smooth variations of centroid displacements is a natural concomitant of the vector-field approach, but requires an extra and arbitrary assumption in the combinatorial approach.

Two other lines of evidence in favor of the vector-field model arise from the distortions of field shape for place cells and overall positional firing pattern distributions for theta cells. Both the existence and precise nature of such “tidal” effects are predicted by the vector-field equation. In contrast, there is no basis from the strictly qualitative combinatorial theory to predict that such effects should exist, and, if so, what form they might take. Incorporating stretching of fields into the combinatorial theory is possible only by specifying the exact nature of the relationships among the triggering stimuli for each place cell. If it turns out necessary to propose that the relationships are the same for all cells or that the relationships vary systematically with position, the combinatorial model will come to resemble the vector-field theory.

In addition to differences from the combinatorial theory, our approach also diverges in several ways from the work of O’Keefe and Burgess (1996). First, O’Keefe and Burgess (1996) focused on the shape of the apparatus, to the exclusion of “marker” stimuli. Our two-card experiments are a new line of evidence that nonstructural aspects of the stimulus environment can be major determinants of firing field activity (see also Sharp et al., 1990; Bostock et al., 1991; Kentros et al., 1998). This is not, however, a fundamental issue; the feature-based theory of O’Keefe and Burgess (1996) can be modified to include marker stimuli as well as walls.

A more important issue is the decision by O’Keefe and Burgess (1996) to identify specific walls of their rectangular (and square) apparatuses as the key features for individual place cells. This approach has the advantage that it provides a direct account of the shape of firing fields as well as of their transformations with changes in the aspect ratio of the rectangle. In this regard, the wall-feature theory resembles strongly the combinatorial theory, although the tendency of nearby walls to exert strong control has a spatial flavor (O’Keefe, 1999). Thus, the wall-feature theory seems to present a quantitative challenge to our views. In reality, however, vector-field theory is not designed to explain the current form of either the wall-feature theory or the aspect-ratio data on which it is based. The key difficulty is that the magnitude of the shape changes used by O’Keefe and Burgess (1996) were large enough that
they induced in some cells major changes in firing pattern shape or suppression of firing fields. In agreement with earlier experiments on doubling the size of an apparatus (Muller and Kubie, 1987), O’Keefe and Burgess (1996) found that the fields of some cells were strongly altered in one or more rectangular apparatuses, putting the data outside of our discourse. We predict, however, that repeating the O’Keefe and Burgess (1996) experiment with smaller (e.g., 20–30%, instead of 100%) changes of wall length would leave all fields recognizable and therefore provide a test of the viability of a modified vector-field approach.

The issue of further testing the vector-field theory raises the additional problem that the form of the theory presented here is extremely specific; it is useful only for two stimuli and for two stimuli inside a cylinder. One direction for future research is to test whether the effects of other stimulus manipulations (e.g., changing the aspect ratio of a rectangle or modifying the cylinder into an elliptically shaped chamber) are also amenable to a vector-field approach. If field centroids move as though the representation of the apparatus floor is again being topologically stretched, it will be important to try to develop more general forms of the vector-field equation. In turn, such a generalization should permit the design of additional experiments to test the model even more rigorously.

Two other extensions of the two-card experiment and its mathematical description are worth considering. First, the two-card experiment serves as a useful (but by no means unique) method of exploring information processing within the hippocampus and its related structures (Sharp, 1999). Thus, it would be extremely interesting to see how reconfigurations would alter the directional firing properties of head-direction cells (Taube et al., 1990; Taube, 1994) and of cells in entorhinal cortex, the dentate gyrus, the subiculum proper, and other parts of the subicular complex. Second, the two-card experiment provides a way of testing whether place cells are in at least some circumstances causally involved in guiding navigational behavior. Specifically, if a rat were trained to find a hidden goal at a certain location in the presence of two cue cards, we expect that the vector-field equation would predict the animal’s new choice of goal location after the cards are reconfigured.

The final point we raise concerns the nature of the neural network that is responsible for place-cell activity and that may be used to guide navigational behavior. In our experiments, every place cell was affected by both stimulus cards if both were present, and removing either card left fields unchanged and under the full control of the remaining card. We therefore chose to develop a numerical theory that emphasizes the global control of all cells by each identified stimulus. Our choice of such a model reflects our belief that the place-cell population represents a large scale, unitary construct, the environment, rather than an agglomeration of separate, smaller-scale features of the environment such as pairwise relationships between objects (O’Keefe and Conway, 1978; Hetherington and Shapiro, 1997; but see Shapiro et al., 1997; Wood et al., 1999). In part, this view is based on the notion that place cells (especially CA3 place cells) interact with each other and are not a set of independent feature detectors (Muller et al., 1991, 1996; Blum and Abbott, 1996; Mehta et al., 1997; Samsonovich and McNaughton, 1997). It is our contention that connections among place cells will prove to be the essential feature that allows the place-cell ensemble to be used in navigation. We speculate that the mutual excitation of place cells may have to be taken into account to explain the position-independent decreases of the firing rates we observed; decreases in the stimulus drive on the place cells may not allow quantitative solutions in a neural network theory.

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