Regulation of Organelle Acidity

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Abstract Intracellular organelles have characteristic pH ranges that are set and maintained by a balance between ion pumps, leaks, and internal ionic equilibria. Previously, a thermodynamic study by Rybak et al. (Rybak, S., F. Lanni, and R. Murphy. 1997. Biophys. J. 73:674–687) identified the key elements involved in pH regulation; however, recent experiments show that cellular compartments are not in thermodynamic equilibrium. We present here a nonequilibrium model of lumenal acidification based on the interplay of ion pumps and channels, the physical properties of the lumenal matrix, and the organelle geometry. The model successfully predicts experimentally measured steady-state and transient pH values and membrane potentials. We conclude that morphological differences among organelles are insufficient to explain the wide range of pHs present in the cell. Using sensitivity analysis, we quantified the influence of pH regulatory elements on the dynamics of acidification. We found that V-ATPase proton pump and proton leak densities are the two parameters that most strongly influence resting pH. Additionally, we modeled the pH response of the Golgi complex to varying external solutions, and our findings suggest that the membrane is permeable to more than one dominant counter ion. From this data, we determined a Golgi complex proton permeability of 8.1 × 10⁻⁶ cm/s. Furthermore, we analyzed the early-to-late transition in the endosomal pathway where Na,K-ATPases have been shown to limit acidification by an entire pH unit. Our model supports the role of the Na,K-ATPase in regulating endosomal pH by affecting the membrane potential. However, experimental data can only be reproduced by (1) positing the existence of a hypothetical voltage-gated chloride channel or (2) that newly formed vesicles have especially high potassium concentrations and small chloride conductance.

Key words: pH regulation • V-ATPase • proton leak • membrane potential

Introduction Intracellular organelles have characteristic luminal pHs suited to their biochemical function. Most of the organelles along the endocytic and secretory pathways, as well as lysosomes, maintain acidic interiors through the action of a primary electrogenic proton pump, the V-ATPase. From Fig. 1 A we see the diverse range of pHs present in the cell. It is our goal to understand the mechanisms that enable organelles to establish and maintain these luminal pHs. In addressing this problem, we hope to elucidate the connection between organelle pH, morphology, and luminal contents. We have constructed a model of acidification based upon specific membrane ion pumps and leaks and the internal ionic equilibria characteristic of the luminal matrix material. Using the model, we have explored the mechanisms of acidification in specific organelles and deduced some general features of pH regulation.

Proton concentration plays a fundamental role in many cellular processes. These roles can be quite different. For instance, osteoclasts use low pH to dissolve bone, whereas neural synaptic vesicles utilize the pH gradient across the bilayer to drive the secondary transport of neurotransmitters (Moriyama and Futai, 1990; Chatterjee et al., 1992). Interestingly, organelles along both the secretory and endocytic pathways experience a gradient of decreasing pH (Cain et al., 1989; Fuchs et al., 1989; Wu et al., 2000). In endosomes, acidification is required for the proper sorting of receptors from ligands. An excellent review of these systems can be found in Futai et al. (2000). Many of the pH regulatory elements appear in all cellular organelles. For instance, proton leaks have been identified in both the exocytic and secretory pathways, whereas V-ATPases appear to be integral components of nearly all organelles (Van Dyke and Belcher, 1994; Kim et al., 1996; Schapiro and Grinstein, 2000; Wu et al., 2000). Endosomes and Golgi complex are permeable to counter ions such as chloride and potassium, which can affect pH by altering the membrane potential (Van Dyke and Belcher, 1994; Schapiro and Grinstein, 2000). Although many regulatory elements are present in most, if not all, organelles, other elements appear to be restricted to particular organelles; e.g., the Na,K-ATPase that limits acidification in the early part of the endosomal pathway (Cain et al., 1989; Fuchs et al., 1989; Teter et al., 1998). Fig. 1 B lists the major el-
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elements that we have considered in our study. Many of the seminal experiments from which this list was compiled will be discussed as we construct the model; for an alternative discussion see Rybak et al. (1997).

Motivation for our model comes from the large number of uncertainties and interrelated effects involved in pH regulation. It is often difficult to extract useful information from even the most precise experiments when the systems involved are as complex as the ones we have examined. Thus, a model based on the physics of membrane biology that employs realistic descriptions of the relevant bioenergetic proteins is useful in precisely interpreting experimental data. Since our model is kinetic, it is able to explain the entire pH curve measured during acidification experiments. Thus, in addition to fitting steady-state data, our model can fit initial slopes and transients. From these fits, we determine the total numbers of pumps present in a particular experiment. In principle, using this analysis to compare different organelles can generate predictions about pump sorting events. While such biochemical information may be present in the experimental data, it requires a comprehensive model to quantify it. Sorting events, and hence their quantification, are critical to understanding pH regulation.

MATERIALS AND METHODS

A Model for Organelle Acidification

The problem of pH regulation largely reduces to regulating the activity of the vacuolar proton pumps (the V-ATPases), which are the primary acidifying agent for most cellular organelles. As an organelle acidifies, a proton motive force (pmf) is built up that opposes further acidification. The transmembrane pmf \( \Delta \mu \) across an organelle bilayer has two components: (1) the membrane potential \( \Delta \psi \), and (2) the proton concentration gradient \( \Delta p \): \[
\text{pmf} = \Delta \mu = \Delta \psi - 2.3 \frac{k_B T}{e} \Delta p \text{H},
\]

where \( k_B \) is Boltzmann’s constant, \( T \) is the absolute temperature, and \( e \) the electronic charge. However, the varying concentrations of all other ions also affect the membrane potential, so that predicting \( \Delta \psi \) requires tracking the movement of the dominant counter ions, the membrane capacitance, as well as the buffering and Donnan properties of the lumenal matrix.

By their effect on the membrane potential, the proton pumping ability of the V-ATPase can be influenced by other ion pumps, such as the Na,K-ATPases as well as chloride and/or potassium channels. In addition, proton leaks and can dissipate gradients that are built up by the proton pump. Moreover, organelle shape and size are important parameters in determining membrane capacitance and, thus, acidification rates. Matters are further complicated by the osmotically driven movement of water in response to changing ion concentrations. To facilitate discussion of the various factors influencing acidification, we shall divide our discussion into three general categories: pumps, channels, and organelle shape and contents.

Ion Pumps

For our purposes, an ion pump can be characterized by its “performance surface” giving the average rate of a single pump as a function of the transmembrane ion gradient and electrical potential, denoted \( J(\Delta \text{pH}, \Delta \psi) \). To measure this surface, a comprehensive series of experiments must be performed. However, since these experiments have not yet been carried out in sufficient detail, we shall rely on models for each pump.

We constructed a mechanochemical model of the V-ATPase that predicts the proton flux as a function of environmental conditions (Grabe et al., 2000). By calibrating the model from whole vacuole patch-clamp experiments the proton pumping performance surface, \( J_{\mu}(\Delta \text{pH}, \Delta \psi) \), was computed (Fig. 2 A).

A similar mechanistic molecular model of the Na,K-ATPase does not yet exist. However, for our purposes, an accurate kinetic description will suffice. These enzymes generally localize to the

1 Abbreviations used in this paper: MVB, multivesicular bodies; pmf, proton motive force; RRC, receptor recycling compartments.
hydrolysis is $21 \text{kBT}$. In principle, the pumping surface can be modified by a pH and membrane potential across the bilayer. The bulk cytoplasm of cells where high turnover rates are needed to maintain low cytoplasmic sodium levels. Many kinetic models describe the pump only under this limited set of environmental conditions and generally neglect reverse reactions necessary for preserving detailed balance (Hartmann and Verkman, 1990; Sagar and Rakowski, 1994). However, the Na,K-ATPases found in organelles may not operate over a much greater set of conditions. Therefore, we shall use the more general model of Apell and Lauger, which obeys microscopic reversibility (Lauger and Apell, 1986). We have combined two extensions of this model that take into account most of the known experimental data on the potassium and sodium portions of the pump cycle (Heyse et al., 1994; Sokolov et al., 1998). From this combined model, the sodium flux ($J_n$) and potassium flux ($J_k$) for an individual ATPase were determined. We have plotted the response surface for the potassium portion of the flux in Fig. 2B. Accurate performance surfaces such as this are sufficient to predict the Na,K-ATPase's response to transmembrane ionic and potential differences. A description of both pump models along with the Matlab™ code used to compute them is given in Online Supplemental Material.

The performance surfaces characterize the average behavior of individual pumps. To compute acidification rates, we must know the number of active pumps. This can be estimated from electron microscopy and biochemical assays (e.g., mAb tagging). However, these estimates are difficult and often inaccurate since only the total number of pumps, not the active fraction, can be deduced from microscopy. Therefore, we shall estimate the number of active pumps in a given organelle by fitting experimental acidification curves or by matching steady-state pH values.

Before continuing, it is natural to ask how heavily our results depend upon the exact shape of the performance surfaces in Fig. 2. To address this, all simulations have been computed with simpler pump models assuming that the pump flux decreases linearly with increasing electrochemical gradient. Most of the data we fit with the nonlinear performance surface can be fit with a linear pump model. However, this necessitates significant changes in key parameters, such as the number of active pumps. Thus, for the systems examined, the exact dependence of the pumps on environmental conditions is not critical. What is important is that the pumps provide a source of ion flux, and pH regulation results from the interplay of this source with other regulatory elements to be discussed. Why then use the nonlinear pumping surfaces in Fig. 2? First, there are situations where the pump mechanism is crucial to pH regulation, such as hyperacidic organelles whose pH is below 2. Second, it is as easy to compute the regulation curves using the nonlinear pump model as with the linear model, and it does not obfuscate the role that pumps play in the overall scheme.

Channels: Chloride, Potassium, and Proton Leak

Intact bilayers are somewhat permeable to protons, but relatively impermeable to other ions. Ion-specific channels allow an organelle to equilibrate specific ions between the lumen and cytoplasm. Movement of these ions through the channel is driven by the transmembrane concentration difference and the membrane potential. The simplest model for this movement assumes that the ion and the channel do not interact during transport. In this limit, the permeability coefficient is the only relevant parameter determining the dynamics of transport. In the presence of a membrane potential, the diffusion flux of ions can be described by Eq. 2 (Hille, 1992; Weiss, 1996):

$$J_i = P_i S \left[ C_i, E_{r,i} - C_i, e^{-z_i U} \right] F / (RT),$$

where $i$ denotes the ionic species (Cl$^-$, K$^+$, or H$^+$), $P_i$ is the permeability of the membrane to ion $i$, $S$ is the surface area of the compartment (C) or lumen (L), $z_i$ is the valence of the ion, and $U$ is the reduced membrane potential, $U = \Psi F / (RT)$. $F$, $R$, and $T$ have their usual meanings.

Proton movement across membranes is not well understood. Protons are permeant to intact bilayers, so it is possible that their transport is not channel mediated. It may involve a more complicated mechanism using water wires or the partitioning of weak

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**Figure 2.** Performance surfaces. (A) The proton pumping rate [$\text{H}^+$/s] for a single V-ATPase is plotted as a function of lumen pH and membrane potential across the bilayer. The bulk cytoplasmic pH is 7.4, but the proton concentration at the membrane is modified by a $-50$-mV surface potential. The free energy of ATP hydrolysis is $21 \text{kBT}$. The pumping profile was computed from the composite model found in Online Supplemental Material. See Table S1 (available at http://www.jgp.org/cgi/content/full/117/4/329/DC1) for a complete list of all parameters.
acids into the bilayer (Paula et al., 1996; Hill et al., 1999). Additionally, voltage-gated proton channels exist in the plasma membrane of certain cell lines, where they are responsible for extruding acid from the cytoplasm (DeCoursey and Cherny, 2000). Whether such channels are functional in organelles is not known. If transport is mediated by voltage-gated channels, the mathematical description above may be inappropriate for the proton flux. Lacking a concrete description of the passive proton flux, we continue with the simple treatment of Eq. 2. As we shall see, this model fits experimental data quite well.

Net proton movement across the membrane is a competition between V-ATPase pumping and channel-mediated leaking. In general, high physiological concentrations of counter ions lead to counter ion movement that is much faster than either of these two processes. This follows from Eq. 2 since the flux is proportional to the difference in ionic concentrations. Thus, no new time scale is introduced into the problem by incorporating channels; the dynamics of acidification will primarily be determined by the proton movement.

We deduce proton permeabilities from data associated with each of the experiments examined. The permeability value for chloride, listed in Table I, is taken from measurements on non-voltage-activated channels residing in the plasma membrane. We shall assume that the permeabilities of the other counter ions have this same value. The rate of acidification is unaffected by variations in the permeabilities as long as the permeability does not become so small as to place counter ion movement on the same time scale with proton movement.

### Table I

<table>
<thead>
<tr>
<th>Description</th>
<th>Units</th>
<th>Symbol</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy of ATP hydrolysis</td>
<td></td>
<td>ΔG_{ATP}</td>
<td>21 kT</td>
<td>Rybak et al., 1997</td>
</tr>
<tr>
<td>Cytoplasmic pH*</td>
<td></td>
<td>pH_c</td>
<td>7.0–7.5</td>
<td>Roos and Boron, 1981; Wu et al., 2000</td>
</tr>
<tr>
<td>Cytoplasmic sodium concentration</td>
<td>mM</td>
<td>Na_c^+</td>
<td>15</td>
<td>Alberts et al., 1994</td>
</tr>
<tr>
<td>Cytoplasmic potassium concentration</td>
<td>mM</td>
<td>K_c^+</td>
<td>140</td>
<td>Alberts et al., 1994</td>
</tr>
<tr>
<td>Cytoplasmic chloride concentration</td>
<td>mM</td>
<td>Cl_c^+</td>
<td>4</td>
<td>Alberts et al., 1994</td>
</tr>
<tr>
<td>Cytoplasmic phosphate concentration</td>
<td>mM</td>
<td>P_c^-</td>
<td>5</td>
<td>Alberts et al., 1994</td>
</tr>
<tr>
<td>Cytoplasmic calcium concentration</td>
<td>M</td>
<td>Ca_c^{2+}</td>
<td>10^{-7}</td>
<td>Alberts et al., 1994</td>
</tr>
<tr>
<td>Extracellular sodium concentration</td>
<td>mM</td>
<td>Na_e^-</td>
<td>145</td>
<td>Alberts et al., 1994; Rybak et al., 1997</td>
</tr>
<tr>
<td>Extracellular potassium concentration</td>
<td>mM</td>
<td>K_e^+</td>
<td>5</td>
<td>Alberts et al., 1994</td>
</tr>
<tr>
<td>Extracellular chloride concentration</td>
<td>mM</td>
<td>Cl_e^-</td>
<td>110</td>
<td>Alberts et al., 1994</td>
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<tr>
<td>Extracellular calcium concentration</td>
<td>mM</td>
<td>Ca_e^{2+}</td>
<td>2.5-5</td>
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<td>Extracellular phosphate concentration</td>
<td>mM</td>
<td>P_e^-</td>
<td>1</td>
<td>Vander et al., 1985</td>
</tr>
<tr>
<td>Bilayer capacitance</td>
<td>μF/cm²</td>
<td>C_o</td>
<td>1</td>
<td>Israelachvili, 1992</td>
</tr>
<tr>
<td>Golgi complex buffering capacity</td>
<td>mM/pH</td>
<td>B_o</td>
<td>10, 40</td>
<td>Farinas and Verkman, 1999; Wu et al., 2000</td>
</tr>
<tr>
<td>ER buffering capacity</td>
<td>mM/pH</td>
<td>ER</td>
<td>6</td>
<td>Wu et al., 2000</td>
</tr>
<tr>
<td>Endosome buffering capacity</td>
<td>mM/pH</td>
<td>EN</td>
<td>50</td>
<td>Rybak et al., 1997</td>
</tr>
<tr>
<td>Chloride permeability</td>
<td>cm/s</td>
<td>P_D</td>
<td>1.2 × 10^{-5}</td>
<td>Hartmann and Verkman, 1990</td>
</tr>
<tr>
<td>Proton permeability</td>
<td>cm/s</td>
<td>P_D</td>
<td>4.8–0.67 × 10^{-5}</td>
<td>Van Dyke and Belcher, 1994</td>
</tr>
<tr>
<td>Golgi complex volume</td>
<td>cm³</td>
<td>V_o</td>
<td>6 × 10^{-12}</td>
<td>Ladinsky et al., 1999</td>
</tr>
<tr>
<td>Endosome volumes</td>
<td>cm³</td>
<td>V_{EN}</td>
<td>88–0.7 × 10^{-15}</td>
<td>Van Dyke and Belcher, 1994</td>
</tr>
<tr>
<td>Golgi complex surface area</td>
<td>cm²</td>
<td>S_o</td>
<td>8 × 10^{-6}</td>
<td>Ladinsky et al., 1999</td>
</tr>
<tr>
<td>Endosome surface area</td>
<td>cm²</td>
<td>S_{EN}</td>
<td>1–0.06 × 10^{-8}</td>
<td>Van Dyke and Belcher, 1994</td>
</tr>
</tbody>
</table>

Names and values of relevant physiological parameters.

*We occasionally refer to pH_c = 7.4 as a typical cytoplasmic pH. This value may appear high in relation to older literature (Roos and Boron, 1981), but it is in line with many of the most recent fluorescence experiments (Wu et al., 2000).

**Organelle Properties: Buffering Capacity, Donnan Potential, Osmotic Effects, and Shape**

The lumenal matrix of an organelle is composed of a polymeric network of proteins. These proteins tend to be negatively charged, and can buffer changes in pH. The negative charge of the matrix is balanced by mobile counter ions which generate a Donnan potential. The volume and surface area of the organelle also play an important role in determining concentrations, membrane potential, and membrane fluxes. These parameters are among the most difficult to quantify primarily because of the large variations in organelle size and shape. Electron micrographs show that even vesicles from purified populations exhibit a broad distribution of volumes and surface areas (Belcher et al., 1987). For the organelles studied here, we have tried to find the most current measurements of these two parameters; a list is given in Table I. When values were uncertain, these parameters were varied to determine the best fit to experimental data.

The buffering capacity, β (units: mol/pH), is the degree with which the luminal matrix is able to bind protons from solution. Since the matrix composition is quite complex, the buffering capacity can only be determined experimentally by biochemical analysis. At a particular luminal pH, this analysis determines the constant of proportionality between the change in proton concentration of the lumen and the change in pH, Δ[H^+] = β · ΔpH. In general, this relation is not linear since β depends on the luminal pH. For pH values near 7, the buffering capacities for the ER and Golgi complex are found to be approximately constant, varying between 6–40 mM/pH unit. Similar experiments...
have been performed on endosomes and lysosomes in which the buffering capacity did not appear to be constant (see Figure 55; Van Dyke, 1993; Van Dyke and Belcher, 1994). For each simulation, the buffering capacity was determined from experimentally measured values. This is discussed in more detail in Online Supplemental Material and the review by Roos and Boron (1981).

The membrane potential affects the flow of ions across lipid membranes and biases the distributions of those ions at steady state. Electroneutrality requires that no net charge exists in any small volume; the membrane potential arises from the microscopic deviation from electroneutrality at a lipid boundary. Physiological models generally exploit the concept of electroneutrality to solve for the membrane potential without detailed information about the electrical makeup of the cell. This requires that the ionic currents crossing the membrane sum to zero at all times. This constraint results in a single algebraic equation whose root gives the membrane potential. Additionally, Hodgkin-Huxley type models relate the electrical activity of the cell to the movement of ions across the membrane by a differential equation for the time dependence of the potential (Hodgkin and Huxley, 1952). Both of these approaches ignore two important features that strongly influence the membrane potential: (1) fixed lumenal charges, and (2) charged lipid headgroups in the bilayer. To include these elements, a physical model of the membrane potential in terms of ionic charge distributions is required.

Poisson-Boltzmann models (more specifically, Gouy-Chapman methods) provide one approach for determining the electrical profile and ion concentrations near the lipid bilayer. To accurately determine these profiles all charged solutes must be included in the calculation (McLaughlin et al., 1981). This is beyond the scope of the present model, since we track only the dominant counter ion concentrations. Motivated by Rybak et al. (1997), we give an explicit form for the membrane potential across the bilayer in terms of the excess charge inside the organelle. We assume that the net charge localizes to the lumenal leaflet, so that we can treat the membrane as a parallel plate capacitor. This is valid since the radius of curvature of organelle surfaces is quite small compared with their thickness. The potential drop across the bilayer is written as Eq. 3:

\[
\Delta \psi = \frac{F}{C_0} \frac{V}{A} \left( \sum \psi_i \cdot [\text{cations}]_i - \sum \psi_i \cdot [\text{anions}]_i \right) + \frac{\beta \cdot \left( \text{pH}_i - \text{pH}_L \right)}{2} - \frac{B}{3}
\]

where \( A \) is the surface area of the membrane, \( C_0 \) is the capacitance per unit area of the membrane (\( A \cdot C_0 \) is the total capacitance of the membrane), \( V \) is the volume of the organelle, \( F \) is Faraday’s constant, and the numbered terms give the concentrations of charged particles are as follows. (1) Sum of ions (chloride, potassium, sodium, and calcium) weighted by their stoichiometric coefficient \( n \) (e.g., \( n = 1 \) for monovalent and \( 2 \) for divalent ions). (2) Total amount of buffered and free protons in the lumen. \( \beta \) is the buffering capacity. We assume that protons do not contribute to the membrane potential when the lumenal pH is equal to the cytoplasmic pH. When the buffering capacity is not constant, this term must be integrated to give the total number of luminal protons contributing to the membrane potential. (3) Molar concentration of all impermeant charges. This term primarily represents fixed negative protein charges trapped in the lumen; however, fixed cations (such as calcium), which keep the luminal matrix condensed, may reduce the magnitude of this term (Verdugo, 1991). Experimental estimates of these concentrations vary by >100 mM from organelle to organelle. Some of these values are listed in Table I. In each simulation, this parameter is estimated from resting pH values in the absence of proton pumping or from best fits as a variable parameter.

Changes in the membrane potential, according to Eq. 3, come about as a direct consequence of changes in the lumenal concentrations of those ions being tracked. The constant charge density term (B) sets the zero point of the membrane potential and biases the lumenal proton concentration as well as the ionic concentrations in term 1. Large values of B have been implicated in the acidic pH that lysosomes maintain in the absence of proton pumps (Moriyama et al., 1992). A recent study using a similar model for the membrane potential has shown good agreement with experimentally predicted membrane potentials (Endresen et al., 2000).

To account for the effects of surface charge, we include a phenomenological potential difference between the bulk cytoplasm and the organelle’s outer leaflet, \( \Delta \psi_{C,0} \) and the bulk lumen and the inner leaflet, \( \Delta \psi_{L,0} \) (Hille, 1992). These potentials modify all ionic concentrations at these surfaces by a Boltzmann factor:

\[
[C_i]_{C,0} = [C_i]_L \cdot \exp \left( \frac{-z_i F \Delta \psi_{C,0}}{RT} \right)
\]

\[
[C_i]_{L,0} = [C_i]_L \cdot \exp \left( \frac{-z_i F \Delta \psi_{L,0}}{RT} \right)
\]

where the surface concentrations are denoted by a zero subscript. Given the typical surface potentials measured for negative phospholipid bilayers bathed in frog Ringer’s solution, this theory predicts Ca\(^{2+}\) surface concentrations to be 20–100 times higher than bulk values (Hille, 1992). These modified concentrations are the values that must be used for computing current flow through membrane channels and pumps. For example, surface concentrations must replace the bulk values in Eq. 2 to solve for the instantaneous flow of ions across the membrane. In general, as the lumen is acidified and ionic concentrations change, \( \Delta \psi_{L,0} \) will also change. We assume that the change in net ionic concentrations is small enough to ignore this effect.

The lipid compositions of cellular membranes are inherently asymmetric, with the cytoplasmic leaflet tending to be more negatively charged due to the preponderance of acidic lipids (Rothman and Lenard, 1977). The surface potential depends strongly on this composition. Vesicles containing 33% acidic lipid in 100 mM monovalent salt concentrations have surface potentials about −50 mV (McLaughlin, 1989). The most relevant physiological measurements of surface potentials have been performed on the plasma membrane. Cahalan and Hall (1982) observed a cytoplasmic surface potential of −30 mV in frog nerve cells. Lacking specific information about the lipid composition of the organelles studied, we assign reasonable, yet arbitrary, values of \( \Delta \psi_{C,0} = −50 \) mV and \( \Delta \psi_{L,0} = 0 \) mV. We retain these values throughout our study and examine the sensitivity of our model to these parameters in Table V. For cytoplasmic surface potentials in the range of −30 to −50 mV, we expect surface concentrations of divalent and monovalent cations to be ~5–50 times higher than bulk concentrations.

Although it is generally accepted that surface charges influence ion concentrations near the membrane, this effect on the operation of pumps and channels is not well characterized. The charges on the transporter itself could affect the local ion concentration much more than the nearby lipids. Additionally, the V-ATPase, like many transporters, is quite large, and the ion binding sites may be more than a Debye length away from the membrane. This issue can be answered only by experiments. For the model trans-
porters used here, we discuss how changes in surface potential affect flux rates in the Online Supplemental Material. There, we show that when the surface potential is zero, predicted permeability coefficients increase by a factor of 2–3, and proton pumping rates decrease by less than a factor of 2. Nevertheless, the overall theme of our discussion remains unchanged.

Experimental measurements of the membrane potential are between the bulk cytoplasm and the bulk lumen. Using the model just outlined, this total potential difference is given by Eq. 5:

$$ \Psi_{\text{total}} = \Delta \Psi + (\Delta \Psi_{\text{c,0}} - \Delta \Psi_{\text{l,0}}) $$  (5)

All reported membrane potentials refer to the total membrane potential. Using the definitions in Eq. 5, \( \Delta \Psi_{\text{total}} > 0 \) will drive anions into the lumen and cations out.

Differences in ion concentrations across the membrane give rise to an osmotic pressure that drives water across the membrane. This effect will change the total volume of the organelle and may influence the lumenal pH. The flux of water into the lumen can be written as Eq. 6:

$$ J_W = P_W \cdot S \left( \sum \frac{\Phi_i}{a_i} [C_i]_L - \Phi_C \right), $$  (6)

where \( \Phi_C \) is the osmolarity of the cytoplasm or external solution and \( P_W \) is the permeability. The Water permeability of endosomes isolated from rat kidney is \( P_W = 0.052 \text{ cm/s} \) (Lencer et al., 1990). For ionic species \( i \), \( \phi_i \) is the osmotic coefficient, \( a_i \) is the solute activity, and \([C_i]_L\) is the lumenal concentration. The osmolarity of the cytoplasm is constant with a typical value of \( \Phi_C = 291 \text{ mM} \). We assume that vesicles are initially in osmotic equilibrium. This allows us to assign an osmolality for those lumenal solutes that are impermeable to the membrane. The osmotic coefficient of the charged solutes is taken as 0.73.

Numerical Solutions

With Eqs. 1–6 we can construct a model organelle and write a set of balanced equations that keep track of the movement of ions across its membrane. A typical set of equations are as follows:

\[
\begin{align*}
(a) \ \frac{dH^+}{dt} & = N_H \cdot J_{H^+} (pH_L, \Psi) - P_{H^+} \cdot S \cdot U(H^+_L - [H^+]_C \cdot e^{-U}) \\
(b) \ \frac{dK^+}{dt} & = -2 \cdot N_{Na,K} \cdot J_{Na,K} (Na_L,K_L,\Psi) - \\
(c) \ \frac{dNa^+}{dt} & = +3 \cdot N_{Na,K} \cdot J_{Na,K} (Na_L,K_L,\Psi) \\
(d) \ \frac{dCl^-}{dt} & = P_{Cl^-} \cdot S \cdot U([Cl^-]_L - [Cl^-]_C \cdot e^{-U}) \\
(e) \ \Psi & = \frac{F}{C_0} \cdot \frac{V}{S} \cdot \\
& \quad \{[Na^+]_L + [K^+]_L \cdot (-[Cl^-]_L + \beta(pH_L) \cdot (pH_L - pH_{K_l}) - B)\} \\
(f) \ \frac{d\rhoH}{dt} & = -\beta(pH_L) \cdot \frac{d[H^+]}{dt}
\end{align*}
\]

The fluxes, \( J_{H^+}, J_{Na,K}, J_{K^+}, \) and the buffer curve, \( \beta \), are imported numerical functions from the pump performance surfaces and the titration data, respectively. As always, brackets denote molar quantities, whereas stand alone symbols represent number of ions. Time varying variables refer to luminal quantities, all of which are coupled by the membrane potential. The equations were solved using Berkeley Madonna with the Rosenbrock algorithm for stiff differential equations (Press, 1997). All simulations start with an initial membrane potential of zero, and negative Donnan particles are masked by an equal concentration of luminal cations. Initial luminal concentrations of counter ions do not affect steady-state values since they are allowed to diffuse until their Nernst potential is zero. In the next section we solve these equations for various experimental configurations.

Online Supplemental Materials

Detailed calculations and further discussion of specific topics are available online at http://www.jgp.org/cgi/content/full/117/4/329/DC1. These include all of the following: V-ATPase and Na,K-ATPase pumping profiles, buffering capacity, and surface potentials. This site also contains the computer program used to compute organelle acidification rates. It is easy to use, and can be tailored by the user for particular experimental situations; it requires the commercial software Berkeley Madonna. The Matlab code used to compute the pumping profiles in Fig. 2 can be downloaded.

RESULTS

Using the model outlined above, we examined experiments performed on reconstituted organelles and on intact cells. Reconstitution experiments make it possible to measure ATP-dependent acidification of organelles that have been incubated in solutions lacking ATP. These experiments give important information on the kinetics of acidification. However, it is not clear that the process of reconstitution does not drastically alter the biochemical makeup of the organelle. Experiments performed on intact cells can also provide kinetic information, and they are much less invasive than reconstitution experiments. In this section we show that our model is in quantitative agreement with reconstitution experiments, and highlight areas of pH regulation that our model can help elucidate. We also examine data recorded from intact cells to address several outstanding questions in pH regulation.

Calibration: The Model Accurately Describes the Acidification of Organelles

It is necessary to confirm the ability of our model to describe acidification before attempting to draw general conclusions from ambiguous data. We begin by analyzing the ATP-dependent acidification of rat kidney endosomes recorded by Shi et al. (1991). Endosomes were reconstituted in a solution devoid of ATP, effectively turning off the V-ATPase. Next, ATP was reintroduced into the external solution and the subsequent acidification of single endosomes was measured. These experiments form a particularly good benchmark because external variables were tightly controlled in the reconstituted medium and the acidification measurements were performed on single endosomes rather than a population. When modeling a single organelle, model parameters...
such as buffering capacity and pump numbers can be interpreted as literal properties of the organelle rather than average properties of a population of organelles.

In Fig. 5 (A and B), we modeled the acidification of an endosome using a constant (Fig. 5 A) and a variable (Fig. 5 B) buffering capacity. The simulation is of a spherical vesicle 163 nm in diameter containing proton pumps and leaks, potassium channels, and trapped negative charges. The experiment (solid line) was performed in a solution of 100 mM potassium, and we assume that potassium is permeant to the membrane. The diameter, number of proton pumps, and proton permeability were varied to give the best fit to the data. At time zero, the lumen was acidified from its initial pH of ~7.4 by turning on the ATP-dependent V-ATPase. In both simulations, the final membrane potentials were ~15 mV, the energetic equivalent of ΔpH ≈ 0.3. This justifies the authors’ assertion that the membrane potential is not appreciable when potassium is free to diffuse across the bilayer.

We investigated two parameters that affect the shape of the acidification curves: buffering capacity and osmotic fluid flow. In Fig. 3 A, we used a constant luminal buffering capacity of 40 mM/pH, as reported by Shi et al. (1991). The buffering curve used in Fig. 3 B was adapted from endosomal data recorded by Van Dyke and Belcher (1994; see Figure S5 available at http://www.jgp.org/cgi/content/full/117/4/329/DC1). At the same volume, the variable buffering capacity gives a noticeably better fit to the data, indicating the importance of buffering in describing pH dynamics. Water channels were also included in the simulation to allow volume change while the surface area remained constant. We assumed that the spherical vesicle was initially in osmotic equilibrium and that the osmolarity of the external medium was 291 mM, a typical cytoplasmic value. From the shapes in Fig. 3 A, we see that endosomal shrinking occurs as buffered protons expel osmotically active potassium ions. The model predicts an 11% decrease in volume, which is in agreement with the authors’ estimate of a decrease of less than 15%. Interestingly, the change in volume during acidification has no effect on the shape of the curve in Fig. 3 A: the curves with and without water exchange are virtually indistinguishable.

The goodness of the fits, coupled with the constraints imposed by the experiments, indicate that the model can accurately represent the physical situation. The buffering capacity plays a dominant role in the dynamics of acidification, and our ability to extract reliable numbers from fits depends upon the fidelity with which the buffering curves are known. We predict shape changes due to osmotic shrinking, but this effect has little influence over the shape of the endosomal acidification curves.

Table II

<table>
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<th>Parameter</th>
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<td>H⁺ permeability</td>
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<td>Volume</td>
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<td>16</td>
</tr>
<tr>
<td>External potassium</td>
<td>mM</td>
<td>100</td>
</tr>
<tr>
<td>External pH</td>
<td></td>
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<tr>
<td>Donnan particles</td>
<td>mM</td>
<td>100</td>
</tr>
<tr>
<td>Buffering capacity</td>
<td>mM/pH</td>
<td>see figure legend</td>
</tr>
</tbody>
</table>

List of parameters used in the simulations in Fig. 3. In column three, left values refer to panel A and right values to panel B.
Regulation of Organelle Acidity

The pump is not working close to thermodynamic equilibrium, which corresponds to $\Delta \text{pmf} = 4.5$ pH units (Grabe et al., 2000). This suggests that the steady-state pH in endosomes, lacking Na,K-ATPases, is the result of a balance between proton pumping and leaking.

The Model Provides a Consistent Theory of pH Gradient and Membrane Potential

A satisfactory theory of pH regulation requires an understanding of both the electrical- and concentration-dependent portions of the pmf across the organelle bilayer. Here, we model experiments that monitor membrane potential in addition to acidification. A similar class of experiments to those of Shi et al. (1991) involves measuring the fluorescence from whole populations of a specific organelle. One acidification curve is measured for the entire population, and it is assumed that this curve represents the dynamics of an average vesicle in that population. Van Dyke and Belcher (1994) have performed a number of experiments on three distinct endosomal populations extracted from normal rat kidney cells. We modeled their acidification data from the receptor recycling compartment (RRC) and multivesicular bodies (MVB) where they report average values for the surface area, volume, and pump leak rate for each of these populations. In addition, they show that the buffering power of the luminal matrix depends strongly upon the value of the internal pH (see Online Supplemental Material available at http://www.jgp.org/cgi/content/full/117/4/329/DC1).

Fig. 4 (A and B) shows the acidification of vesicles in the presence of chloride and potassium channels, fixed negative luminal proteins, and proton pumps and leaks. For this calculation, we used the buffering curves from Figure S5 (available at http://www.jgp.org/cgi/content/full/117/4/329/DC1); other parameters for these simulations and the percent deviation from measured values are given in Table III. As in the Verkman experiments, large proton permeabilities, on the order of $10^{-3}$ cm/s, suggest that the ratio of the leak rate to the pump rate is the major determinant of steady-state pH. Table III shows that the model parameters are comparable to the average experimental values. The RRC parameters are consistently closer than the MVB parameters to the measured values. However, the most inaccurate MVB parameter (volume) is only a factor of 1.7 larger than the reported value.

To investigate the role of counter ions, Van Dyke and Belcher (1994) varied the external chloride concentration and measured the resulting steady-state membrane potential. When the membrane is permeable to chloride, Eq. 2 predicts that changes in the external chloride concentration will affect the steady-state distribution of the ion across the membrane. Subsequently, the membrane potential will change according to Eq. 3. Fig. 4 (C and D) shows that the model describes the membrane potential’s dependence on the external chloride concentration.

Figure 4. (Top) Modeling the ATP-dependent acidification of vesicles in rat liver from two different stages of endocytosis. Model predictions are shown in solid lines and the experimental data points were provided by Van Dyke and Belcher (1994). Controls include the following: V-ATPases, proton leaks, passive chloride channels, passive potassium channels, Donnan equilibrium, and buffering capacity. See Table III for all parameters. (A) The acidification of the receptor recycling compartment (RRC). The buffering capacity is extrapolated from experimental values determined by Van Dyke and Belcher. (B) Acidification of the much larger multivesicular bodies (MVB). Fitting the data required a pump density on the MVB vesicles approximately eight times greater than the RRC vesicles. (Bottom) Steady-state membrane potential as a function of external chloride concentration. The presence of chloride enhances acidification by allowing chloride to enter the vesicle to reduce the membrane potential against which the pump must operate. Both C and D show that increasing the external chloride concentration is an effective way to reduce the resting membrane potential. (C) For the RRC population, the dependence of the membrane potential on the external chloride concentration matches experimental observations with the same parameters for the simulations in A. (D) A fivefold decrease in the number of proton pumps relative to B is required to describe the MVB population’s dependence on the external chloride concentration.
chloride concentration quite well. As the chloride concentration is increased, a greater luminal concentration of chloride counter balances the presence of protons, thus reducing the membrane potential.

One set of parameters described both sets of RRC data, yet the MVB data required an eightfold decrease in the number of proton pumps to describe the dependence of membrane potential on external chloride concentration. Whether such variations are to be expected from trial to trial is not clear. Interestingly, panels A, C, and D of Fig. 4 have very similar pump densities (~50 pumps/µm²) indicating that no active sorting of the proton pump occurs between these populations. However, the MVB simulation in Fig. 4 B required a pump density of 270 pumps/µm². More consistent fits that agree over more data sets are required before we can make any claim about sorting events. Fig. 4 (C and D) shows that our model quantitatively describes the dependence of the membrane potential on external ion concentrations. The ability to fit both the membrane potential and acidification data with the same set of parameters in Fig. 4 (A and C) suggests that the model provides a consistent theory of both components of the pmf.

**Morphological Changes Alone Cannot Account for the Decrease in pH along the Secretory Pathway**

In the absence of proton leaks, the steady-state pH of a model vesicle depends strongly on its shape and size (Rybak et al., 1997). This dependence can be seen from Eq. 3 in which the membrane potential is proportional to the ratio of the volume to the surface area. When this ratio is small, we expect to see a reduction in membrane potential and lower steady-state pH values. Can this effect alone explain the decrease in pH along the secretory pathway?

We have not been able to find data to directly model this question. However, we note that many experiments performed on intact cells have shown that significant proton leakage occurs in many major organelles (Farias et al., 1999; Schapiro et al., 2000; Wu et al., 2000). Therefore, when revisiting the analyses of Rybak et al. (1997) we included a proton leak when exploring the effects of vesicle shape and size.

In Fig. 5, we plotted the steady-state pH of a nonequilibrium system containing a constant density of proton leaks and proton pumps. The final pH is plotted as a function of surface area and the logarithm of volume. In the bottom panel of Fig. 5, numbers indicate experimentally measured surface areas and volumes of several different types of organelles. In the absence (Fig. 5, top) and presence (Fig. 5, bottom panel) of chloride channels, there is very little change in the final pH as the shape changes from small spherical endosomes and secretory granules (near population 1) to large, floppy Golgi complex (population 3). Both panels exhibit pH changes of one unit over the range of physiological shapes, as also reported by Rybak et al. (1997). However, this effect would only explain pH variations in the cell if small vesicles and endosomes had much more surface area than is seen in electron micrographs.

The model predicts when proton leaks are present, changes in volume and surface area alone cannot account for the wide range of pHs seen in cellular organelles. This suggests that some other mechanism,

---

**Table III**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Panels A and C</th>
<th>RRC</th>
<th>Panels B and D</th>
<th>MVB</th>
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<td>2.0 x 10⁻³</td>
<td>-58</td>
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<td>Surface</td>
<td>cm²</td>
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<td>13.6 x 10⁻⁹</td>
<td>32</td>
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<tr>
<td>Volume</td>
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<td>0</td>
<td>15 x 10⁻¹⁴</td>
<td>70</td>
</tr>
<tr>
<td>Donnan particles</td>
<td>mM</td>
<td>7</td>
<td>-</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>Buffering capacity</td>
<td>mM/pH</td>
<td>see Figure S5</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

List of parameters used in the simulations in Fig. 4. Simulations in panels A and B were performed with 10 mM bulk chloride and potassium, whereas the potassium was 140 mM in panels C and D. The number of V-ATPase proton pumps in panels A and C was 2, whereas in panels B and D had 354 and 70 pumps, respectively. The columns RRC and MVB are the percent differences between model parameters and the average values recorded by Van Dyke and Belcher (1994). The external pH was held fixed at 7.0 for all simulations.
such as the sorting of proton pumps or proton leaks, is the dominant factor in regulating luminal pH.

**Permeability to Multiple Counter Ions Decreases Changes in Golgi Complex pH**

Our goal was to apply the model to the data recorded from intact cells to predict Golgi complex proton permeability values and explore the role of counter ions in regulating Golgi complex pH. Through the biochemical manipulation of intact and membrane permeabilized cells, Wu et al. (2000) were able to affect intracellular conditions and record subsequent changes in organelle pHs. Thus, they are able to carry out experiments similar to the reconstitution studies discussed above. We modeled two different experiments aimed at understanding proton leak and counter ion regulation. First, in the presence of bafilomycin, a potent inhibitor of V-ATPase, the cytoplasm was acid loaded by externally pulsing membrane permeable NH$_4$ under sodium-free conditions. The absence of sodium inhibits Na$^+$/H$^+$ exchange across the plasma membrane thereby keeping the cytoplasm acidic. After NH$_4$ washout, sodium was reintroduced, and the alkalinization of the cytoplasm and Golgi complex was measured over time. Since these experiments lack functional proton pumps, the proton permeability can be determined from the rate of alkalinization. Second, the role of counter ions was investigated in membrane permeabilized cells in Ringer’s solution. The pH of the Golgi complex was recorded as the external solution was replaced with a chloride-free Ringer’s solution.

The experimental information available is too sparse to let us address the dynamic nature of most organelles. Therefore, in the following analysis, we ignore the dynamic nature of the endomembranes. For example, we assume constant volumes, surface areas, and numbers of active pumps. We also ignore the biochemical differences between the trans, medial, and cis portions of the Golgi complex. This simplification is dictated by the fact that pH-sensitive dyes localized to the Golgi complex are spread throughout the entire network, so that the experimental recordings that we model are already averaged over the entire compartment.

In Fig. 6, we examined both the counter ion regulation (main graph) and proton leak (inset) experiments. The determination of the Golgi complex proton permeability came from the experiment in the inset. Since proton pumping was experimentally inhibited, the simulation lacks V-ATPases. At time zero, the cytoplasmic pH begins to alkalinize from the reintroduction of sodium to the external solution (Fig. 6, black circles). The Golgi complex pH rises in response to the change in proton gradient across the Golgi complex membrane (Fig. 6, open squares). Modeling the Golgi complex pH, we are able to predict the rate of alkalinization of the Golgi complex in response to the increase in cytoplasmic pH (Fig. 6, solid line). The Golgi complex leak is modeled well by a simple passive channel. Using the volume and surface area measured by Ladinsky et al. (1999), we predict a proton permeability of 8.1 $\times$ 10$^{-6}$ cm/s. This value for the proton permeability is used in the main figure to examine chloride counter ion effects.

The external chloride analysis discussed above demonstrated that cytoplasmic counter ion concentrations can have dramatic effects on the membrane potential.
and, consequently, on the luminal pH. The main graph of Fig. 6 shows the resting pH of a Golgi complex in a plasma membrane compromised cell. At time zero, the external solution is replaced by a chloride-free Ringer’s solution, effectively decreasing the cytoplasmic chloride from 20 to 0 mM. From our analysis in Fig. 4 (C and D), we expect the Golgi complex membrane potential to increase. After this increase in pmf, the V-ATPase pumping rate should slow, causing the lumen to alkalize, yet experimentally the pH remains unchanged (Fig. 6, black circles). This can be explained in two ways. The Golgi complex is either impermeable to chloride or permeable to chloride and at least one other dominant counter ion. Using the model, we explored the latter possibility. Three separate simulations with differing levels of cytoplasmic counter ion concentration were fit to the data. In the presence of only 1 mM additional counter ion, we predict a noticeable alkalization of the Golgi complex, $\Delta \text{pH} > 0.5$ (Fig. 6, dotted line). The presence of a 14-mM cytoplasmic concentration counter ion (Fig. 6, dashed line) strongly suppresses changes in the luminal pH upon removal of chloride. However, a counter ion concentration of at least 140 mM (Fig. 6, solid line) is required to eliminate any noticeable change in the Golgi complex pH.

Our analysis suggests that if the Golgi complex membrane is permeable to chloride ions, then it is also permeable to potassium. This conclusion is consistent with recent Golgi complex experiments (Schapiro and Grinstein, 2000). The fidelity with which the model reproduces the experimental data suggests that our treatment of the regulatory elements present in the Golgi complex is accurate. We predict the proton permeability of the Golgi complex to be $8.1 \times 10^{-6}$ cm/s. However, this value depends upon many organelle characteristics that are difficult to quantify, such as volume, surface area, and bilayer surface potential.

Can the Na,K-ATPase Limit Acidification in the Early Endosome through Increasing the Membrane Potential?

Endosomes maintain acidic internal pHs to ensure proper sorting of receptors and ligands during endocytosis. Two subpopulations of endosomes can be identified based on their function and capacity for acidification. Early endosomes are responsible for the rapid recycling of receptors, whereas late endosomes are required for transporting cargo onto lysosomes. Late endosomes maintain more acidic lumen than early endosomes. The use of ouabain, an inhibitor of Na,K-ATPase, enhances the acidification of early endosomes both in vivo and in vitro (Cain et al., 1989; Fuchs et al., 1989). The electrogenic Na,K-ATPase is thought to inhibit acidification by increasing the membrane potential across the bilayer. A greater membrane potential, and hence greater pmf, decreases the proton pumping rate of the V-ATPase, which results in a more alkaline lumen. Rybak et al. (1997) have attempted to quantify this hypothesis by computing equilibrium solutions of vesicles containing V-ATPases, Na,K-ATPases, and many of the regulatory elements considered here. We take this analysis one step further and use our kinetic model to simulate the dynamics of acidification in an attempt to shed more light on the feasibility of this proposed mechanism. It should be noted that some endosomes appear to be insensitive to ouabain (Teter et al., 1998). Thus, the mechanism we now examine may not be a universal property of all cell lines.

Cain et al. (1989) have measured the acidification of endosomes in single intact cells both in the presence and absence of ouabain. Initially, cells are bathed in a medium containing fluorescent dye at temperatures prohibitive to endocytosis. The bath is quickly warmed to let endocytosis begin. The total integrated fluorescence of the dye taken up by the newly formed endosomes is measured, and the acidification curves are determined. These acidification curves are averaged recordings from the entire endosomal population as it undergoes all stages of endocytosis. The early measurements are presumably dominated by those endosomes undergoing early endocytosis, and it is in this early portion in which the effects of ouabain are particularly acute.

In Fig. 7, we modeled the initial acidification of the entire endosomal population with a single average spherical vesicle. All simulations included buffering capacity, chloride and potassium channels, V-ATPases, and proton leak. The initial pH of the vesicle was that of the bathing solution, and all internal ion concentrations were assigned standard extracellular values from Table I. The middle dashed curve is the simulation in the presence of Na,K-ATPases, whereas the bottom curve is the same simulation in the absence of Na,K-ATPases. We sought a single set of parameters that could simulate the dynamics of acidification both in the presence (Fig. 7, circles) and absence of ouabain (Fig. 7, diamonds) simply by turning off the Na,K-ATPase. From the two lower curves, we see that our model could not fit both sets of data with this constraint. In the presence of chloride and potassium counter ion conductance, it is very difficult for the Na,K-ATPase to increase the membrane potential enough to limit acidification by more than half a pH unit.

From our studies, the Na,K-ATPase is able to limit acidification by a whole pH unit only (1) when the endomembrane no longer conducts chloride above a certain critical membrane potential, or (2) when the initial internal potassium concentration is far greater than typical external values and there is no chloride conductance. To explore the former hypothesis, we replaced the passive chloride channel, described by Eq. 2, with a voltage-gated channel that has zero conductance when
the curves best modeled by a single endosome. We have shaded the graph at later times to highlight the portion of the endosomal population has begun acidifying near time zero. We smoother since only a small fraction of the what will be the entire pH is present at time zero. The experimental points are endosome begins acidifying at a single instance, hence, a kink in the Data points have been adapted from Cain et al. (1989). The model contains the experimentally measured full pH difference (top curve).

Men anal pH is achieved by incorporating Na,K-ATPases into the vesicle, and 40 mM/(pH unit) buffering capacity. The initial lumenal concentrations of sodium and potassium were assumed equal to standard extracellular values (Table I). A significant increase in lumenal pH is achieved by incorporating Na,K-ATPases into the vesicle (dashed curve). However, the addition of a voltage-gated chloride channel, which is inactivated above 133 mV, is necessary to obtain the experimentally measured full pH difference (top curve). Data points have been adapted from Cain et al. (1989). The model endosome begins acidifying at a single instance, hence, a kink in the pH is present at time zero. The experimental points are smoother since only a small fraction of the what will be the entire endosomal population has begun acidifying near time zero. We have shaded the graph at later times to highlight the portion of the curves best modeled by a single endosome.

In the presence of such a voltage-gated chloride channel, the middle dashed line becomes the top solid curve (Fig. 7), whereas the simulation in the absence of Na,K-ATPases remains unchanged from the bottom curve. Without Na,K-ATPases, the steady-state membrane potential is below the switching voltage of the activated chloride channels, and the chloride counter ions mask the movement of protons into the lumen. The addition of Na,K-ATPases pushes the membrane potential above the switching voltage of the chloride channel, and the chloride ions no longer act as counter ions. As the pumps continue to push protons into the lumen, the membrane potential quickly increases. This adversely affects the V-ATPase pumping rate resulting in a much more alkaline lumen.

From the two solid curves in Fig. 7, we see that the presence of a voltage-gated chloride channel, first proposed in this context by Rybak et al. (1997), greatly increases the ability of the Na,K-ATPase to affect lumenal pH. ClC-5 is a chloride channel that localizes to endosomes in proximal tubule cells; it is a likely candidate for the chloride conductance in the experiments examined here (Günther et al., 1998). This protein belongs to the ClC family of chloride channels that exhibit widely varying current-voltage dependencies (Maduke et al., 2000). For inside positive voltages, ClC-5 channels carry a small inward flux of chloride; however, this flux does not vanish for total membrane potentials >83 mV as we hypothesize above (Friedrick et al., 1999).

Without such a gated chloride channel, or the presence of a high initial potassium concentration in the endosome coupled with no chloride conductance, our model predicts that the Na,K-ATPase can alkalinize the lumen by half of a pH unit, at most.

**Lumenal pH Is Most Sensitive to Changes in Pump and Leak Densities**

We have reviewed many experiments that attempt to address the importance of particular regulatory elements in establishing and maintaining pH. Our model includes many of these elements, and it is important to determine the relative sensitivity of each of these regulatory factors. In principle, such analysis will help experimenters determine which parameters are most critical to the type of questions they want to answer. We chose to revisit the Golgi complex simulations in Fig. 6 to carry out this sensitivity analysis.

The acidification of a model organelle was simulated from an initial lumenal pH of 7.4 until steady state was reached. All relevant parameters were varied ±10% from the original values given in Table IV. As in Fig. 5, the proton pump and leak densities remain constant when the surface area is changed. The difference between the curves generated with and without variation were characterized in two ways: (1) the percentage change in the total pH gradient across the membrane (%ΔpH), and (2) the percent change in the half-time to acidification (%τ). The list of varied parameters is given in Table V, along with the values for %ΔpH and %τ.

From the top two entries in Table V, we see that the most effective ways to change the steady-state pH are to vary the density of active proton pumps or proton leaks.

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**Figure 7.** Endosomal acidification in the selective presence (− ouabain) or absence (+ ouabain) of Na,K-ATPase. We model the acidification of a single 163-nm-diam spherical endosome in the presence (top curves) or absence (bottom line) of 350 Na,K-ATPases. All three simulations include the following: 16 V-ATPases, a proton leak with a permeability of 8.24 × 10⁻⁵ cm/s, chloride channels, passive potassium channels, 140 mM of Donnan particles, and 40 mM/(pH unit) buffering capacity. The initial lumenal concentrations of sodium and potassium were assumed equal to standard extracellular values (Table I). A significant increase in lumenal pH is achieved by incorporating Na,K-ATPases into the vesicle (dashed curve). However, the addition of a voltage-gated chloride channel, which is inactivated above 133 mV, is necessary to obtain the experimentally measured full pH difference (top curve). Data points have been adapted from Cain et al. (1989). The model endosome begins acidifying at a single instance, hence, a kink in the pH is present at time zero. The experimental points are smoother since only a small fraction of the what will be the entire endosomal population has begun acidifying near time zero. We have shaded the graph at later times to highlight the portion of the curves best modeled by a single endosome.

---

**Table IV**

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List of parameters used in the simulations in Fig. 6.
The model provides a method to explore different mechanisms in comparison with experimental data focusing on the most sensitive parameters.

**DISCUSSION**

We have constructed a quantitative model for organelle acidification that can be tailored to address many of the organelles along the secretory and endosomal pathways. Thus, the model can provide a general framework for understanding acidification. By using physically accurate descriptions of but a handful of pH regulatory elements, we have been able to successfully model the acidification of several organelles. The close attention paid to the biochemical makeup of the systems studied eliminated some of the extreme pH and membrane potential values predicted in the earlier work of Rybak et al. (1997). We have presented the first estimates of Golgi complex proton permeability, $\sim 10^{-5}$ cm/s, as well as the first predictions of the active ATPase pump numbers for the Golgi complex and endosome. By modeling experimental data, we were able to make hypotheses about the regulatory elements present in specific organelles. Our model suggests that the Golgi complex is permeable to at least two dominant counter ions. In the endosome, we conclude that it is possible for the Na,K-ATPase to limit lumenal acidification by increasing the membrane potential. However, the presence of voltage-gated chloride channels or extreme ionic conditions are required to explain the high degree with which the Na,K-ATPase has been shown to limit acidification.

The greatest asset of our model lies in its ability to quantify and compare experimental data; however, a good deal of our analysis has been spent justifying the accuracy of our approach. We have not addressed what biochemical differences are required to elicit pH changes between organelles. Secretory granules that bud from the Golgi complex at pH $\approx 6.4$ eventually achieve lumenal pHs of $\sim 5.4$. Table V suggests that changes in the proton pump and leak density are most effective in bringing about this 1-pH unit drop. Calculations were performed on a 200-nm-diam vesicle with properties similar to the Golgi complex simulation of Fig. 6. A 10-fold increase in the proton pump density or a 10-fold decrease in the proton permeability proved sufficient to describe this drop in lumenal pH. Ultimately, we hope that analyses such as these, in conjunction with experimental data, will help elucidate the nature of pH regulation in the cell.

It is interesting to note the discrepancy in proton permeabilities between reconstitution and intact cell experiments. Proton permeability values deduced from reconstitution experiments are $\sim 100$ times higher than values deduced from data recorded from intact cells. It is possible that the process of reconstitution artificially increases the proton permeability of the membrane.
Extending the model to describe additional organelles can be accomplished easily by including the specialized proteins that make these systems unique; e.g., synaptic vesicles that use the proton gradient to drive the uptake of neurotransmitter. This is the approach we used in modeling the early-to-late endosome transition, which required a detailed model of the Na,K-ATPase. Some of the most acidic organelles, such as lysosomes and the central vacuoles of plant cells, have not been discussed. Lemon vacuoles have been shown to acidify to pH 2.5, whereas giant blood cells of the marine Acidian have single large vacuoles with luminal pHs below 1 (Muller et al., 1996; Futai et al., 1998). It is not clear how the V-ATPase could be solely responsible for creating such acidic environments since it is generally thought to be thermodynamically limited to a maximum pH difference of 4.5 units. It is possible that another enzyme contributes to acidification of these systems or that other regulatory elements conspire to create a very negative membrane potential that helps to drive protons into the lumen. Motivated by the presence of V-ATPases in lemon vacuoles, we previously proposed a mechanism by which the V-ATPase proton pump could "change gears" and pump below pH 1 (Grabe et al., 2000). In Online Supplemental Material (see http://www.jgp.org/cgi/content/full/117/4/329/DC1), we describe the molecular mechanism of this gear change and include a calculation confirming that this hypothetical scheme can indeed produce large pH gradients.

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Detailed Calculations and Discussion

Michael Grabe and George Oster

V-ATPase Model

We have constructed a mechanochemical model for the V-ATPase that predicts proton pumping rates over a wide range of environmental conditions. We have used this model to determine the acidification of organelles. The conventional paradigm for active transmembrane ion transport is the “alternating access” mechanism: ions are bound tightly on the low concentration side of the membrane and a conformational change exposes them to the high concentration side and weakens their binding affinity so that they dissociate. The pump then resets its conformation to repeat the cycle (Eisenberg and Hill, 1985; Alberts et al., 1994). The energy to drive the cycle is supplied by nucleotide hydrolysis or an ion gradient. The Na-K-ATPase conforms to this traditional view of alternating access (see below); however, the conformational change in the V-ATPase is a simple rotation. In our computations here, we assume that the V-ATPase is working under normal operating conditions and that ATP concentrations are sufficiently high that hydrolysis is not rate limiting.

By analogy with the F-ATPases, the V-ATPase structure is conventionally divided into a counter-rotating “stator” and “rotor” (Boekema et al., 1997; Finbow and Harrison, 1997; Forgac, 2000). Hydrolysis of ATP in the V₁-soluble headpiece provides the torque that rotates the membrane-bound section V₀. Two models have been suggested for the rotor–stator assemblies: a two-channel model and a one-channel model (Vik and Antonio, 1994; Junge et al., 1997; Elston et al., 1998; Grabe et al., 2000). They differ in the path that the protons take through the enzyme and in the degree to which protons bound to the rotor section communicate with the cytoplasm. There is significant debate about the true structure of the channel in the V- and F-ATPase enzymes. We have shown that both channels behave similarly within the context of our mechanochemical model (Dimroth et al., 1999; Grabe et al., 2000). More recently, experiments on sodium V-ATPases strongly support the one channel model (Murata et al., 2000). There-

![Figure S1. Perspective and face-on views of the rotor–stator assembly showing the paths protons follow in moving from the cytoplasm (bottom) into the lumen (top). Torque supplied by V₁ from ATP hydrolysis turns the rotor to the right. A single half channel penetrates the stator to the level of the rotor sites. A horizontal polar strip connects the right side of the channel and the cytoplasm to allow the passage of an unprotonated site, but protons are blocked from leaking by the stator charge. All other parts of the rotor–stator interface are hydrophobic. Cytoplasmic protons on the left bind to a rotor site, largely neutralizing it. Rotation carries the protonated site across the hydrophobic interface where the site enters the output (acidic) channel (from the left). The stator charges force the site to relinquish its proton into the lumen. The unprotonated site exits the interface to the right and is again in equilibrium with the cytoplasmic reservoir. Note that the sizes of the rotor and stator are such that two rotor sites cannot fit in the rotor–stator interface at once.](image-url)
Therefore, in Fig. S1 we present the putative structure for the one-channel model that was used to compute the V-ATPase performance surface in Fig. 2 A of the text.

The structure in Fig. S1 must possess a few key properties to be a physically viable model. The interface between rotor and stator must present a hydrophobic barrier to prevent proton leakage between the reservoirs. When not in the stator interface, the six rotor acidic sites exchange protons with the cytoplasm. Finally, the output (acidic) channel extends into the stator to the level of the rotor proton binding sites to allow exchange with the lumen.

The one-channel proton pump model works according to the following sequence of events (shown in Fig. S2 A). Rotor sites are in equilibrium with the basic reservoir. Until a rotor site is protonated, the torque generated by $V_1$ cannot force it over the hydrophobic barrier. However, once neutralized by protonation, the torque from $V_1$ drives the site across the barrier into the stator output channel. A protonated site that rotates out of the channel into the polar strip interacts electrostatically with the stator charge, reducing its $pK_a$ and forcing it to relinquish its proton to the acidic reservoir. Further rotation carries the rotor site out of the stator interface, where it quickly equilibrates with the basic reservoir. The full mathematical formulation is contained in the original paper (Grabe et al., 2000).

The pumping cycle in Fig. S2 A does not always culminate in the proton exiting through the output channel. Rather, the protonated site “slips” past the stator charge and reequilibrates with the input reservoir. Fig. S2 B out-

**Figure S2.** Mechanistic description of a typical pumping event and slip. (A) A typical sequence of events following a site as it passes through the rotor–stator interface. An empty site in the basic channel is reflected by the dielectric boundary until a proton binds, neutralizing the site (1 → 2). The $V_1$ motor rotates a protonated site out of the membrane, across the hydrophobic interface, and into the acidic channel (2 → 3). In the channel, the site has a high probability of staying protonated since $k_{on}$ is large. However, when the protonated site rotates close to the stator charge, the rotor site $pK_a$ decreases so that $k_{off}$ increases until the proton is relinquished to the acidic reservoir (3 → 4). The empty site is then driven through the hydrophilic strip, past the stator charge, and into the basic reservoir (4 → 5). The torque from $V_1$ can then rotate the site through an entire revolution back to position (1) where the cycle repeats. (B) The possible pathways for protons passing through the stator. A fraction of the protons that rotate into the output channel slip past the stator charge and dissociate back into the basic reservoir. The proton flux ($J_{IN}$) traverses the hydrophobic interface from the left. A fraction ($J_{OUT}$) dissociates into the output channel, and a fraction ($J_{SLIP}$) slips past the stator charge and leaves the input channel; thus $J_{SI} = J_{IN} - J_{OUT}$. Actually, a site traversing the output channel will bind and dissociate a proton $\sim 10^4 - 10^5 \times$, so the proton that slips back into the basic reservoir is unlikely to be the same one that initially entered on the rotor site (Grabe et al., 2000). Additionally, the back and forth diffusive motion of the rotor may shuttle protons from the output channel to the basic reservoir, creating a second contribution to the slip flux ($J_{S2}$). Thus, the total slip flux is $J_{SLIP} = J_{SI} + J_{S2}$. This second component of the slip flux is only important when the $V_1$ motor is nearly at stall; i.e., when the rotation rate of $V_o$ is small.
lines various slip mechanisms. Slip can have a profound influence upon acidification since it limits the pH that a vesicle can attain. As the pmf across the membrane increases, the enzyme is more prone to slip. The degree of slip can only be determined experimentally from measuring ATP hydrolysis rates and proton pumping. The surface in Fig. S2 A is partially calibrated from patch-clamp experiments and largely free of slip over the range of plotted pmf’s. Excellent reviews of this enzyme can be found in the February special edition of the *J. Bioenerg. Biomembr.* (2000).

The “Gear Change” Model for Hyperacidification

It is generally accepted that the V₃ subunit has three ATP hydrolyzing sites, while the Vₒ rotor has six proton binding sites on each of its proteolipid subunits (c, c’, and c”) that span the lipid bilayer. Under optimal conditions (i.e., no slip), every rotation of the enzyme transports six protons at the expense of three ATP’s, corresponding to a coupling ratio of two. If the six proton binding sites have different pKₐ’s (for instance, three with low and three with high values), exposure to a very acidic luminal environment would permanently titrate the high pKₐ rotor sites. Then the number of active rotor sites would be reduced from six to three, thus decreasing the coupling ratio from two to one. With this coupling ratio, the energy of each ATP would transport one proton, and the maximum achievable pH gradient would double from 4.5 to 9. This may help account for the phenomenon of very acidic organelles. We have revisited the V-ATPase model above, assuming that the six proton binding sites are not equivalent. The sites are assumed to alternate between tightly binding, pKₐ ~ 7.3, and weakly binding, pKₐ ~ 3.8. We also assume that the tightly binding sites hold the proton much closer to the proton-accepting amino acid, creating a much smaller dipole electric field. The existence of more than one proteolipid subunit in the V-ATPase could explain this heterogeneity in proton binding sites. Although all three subunits (c, c’, and c”) are homologous, subtle differences in their proton binding carboxyl groups could account for the variations in proton binding affinities (Forgac, 1999).

In Fig. S3, the proton flux is plotted as a function of luminal pH. The list of simulation parameters are given in Table SI. At modest luminal pH values, pH ~ 6–7, all sites participate in proton transport; however, as luminal pH drops (pH ~ 3.5–6), the high pKₐ sites no longer deliver protons to the lumen. On average, these sites do no work against the pH gradient, thereby enabling the low pKₐ sites to use a larger fraction of the energy of ATP hydrolysis and continue pumping protons. This proposed ability of the V-ATPase to sense and adjust to the absolute pH of the lumen is theoretically interesting. In more alkaline organelles, the V-ATPase works far from thermodynamic stall, where all six proton binding sites would participate in pumping in order to counter the high leak rate of protons. However, in the very acidic organelles, where proton permeabilities are likely to be small, the overall rate of proton pumping should be less important than the size of the pH gradient the enzyme can create. Therefore, as an organelle acidifies, the loss of some of the proton transporting sites is more than made up for by the enzyme’s ability to continue working to produce a low luminal pH.

These calculations are carried out in program files: v_pump.m (main program); cal_2.m; vel_a.m; v_param.m; barrier.m; delc_cf.m; pot.m; pot2.m.

![Figure S3](image-url)

**Figure S3.** The proton flux of a single V-ATPase in the absence of membrane potential is plotted as a function of luminal pH. It is generally accepted that the rotor subunit of the V-ATPase is composed of six proton binding sites. Here we explore the possibility that these sites have alternating high and low pKₐ’s. At high pH values, all sites participate in proton transport from the cytoplasm to the lumen. As the pH drops, the high pKₐ sites begin to leak protons, and then eventually stop participating in transport. This effectively lowers the number of protons transported to ATPs consumed per revolution; e.g., the coupling ratio. Lowering the coupling ratio makes it possible to achieve very acidic luminal pHs. See Table SI for a list of parameters.
Na-K ATPase Model

We have attempted to construct a more general model of the Na-K-ATPase pumping rate over a large range of environmental conditions. This was accomplished by simulating the detailed reaction schemes devised by Heyse et al. (1994) and Sokolov et al. (1998) while constraining the current to obey the voltage dependence measured by Sagar and Rakowski (1994). The model presented here is an amalgam of the most recent Apell model (Heyse et al., 1994), which focuses on sodium transport, with an older model that includes potassium transport. The kinetic scheme can be found in Fig. S4 A. The primary pumping cycle of this model, indicated by solid lines, closely resembles the standard Post-Albers scheme (Heyse et al., 1994). In addition, there are three escapements that involve incomplete pumping cycles. These closed loops are represented by dashed lines and are essential for describing the pump’s behavior under extreme conditions such as low extracellular potassium concentrations.

Steady state values were computed by constructing the kinetic equations corresponding to the reaction scheme in Fig. S4 A and simulating their time evolution until steady state was reached. Alternatively, setting these equations equal to zero results in a set of linear coupled equations that can be solved algebraically to find the steady state values. For ease of implementation, we chose the former. The rate of ion transport was determined in the usual way by summing the forward and backward jump rates between those states that correspond to ion movement from the enzyme to the bulk cytoplasm (or the extracellular space).

To accurately describe pumping under physiological conditions, model parameters have been varied to obtain best fits to the current–voltage data measured by Sagar and Rakowski (1994). Four reverse rates along the potassium portion of the reaction cycle have been adjusted. Additionally, the reverse rates of four more parameters were modified to ensure detailed balance around each of the independent pump cycles. These parameters are indicated in Table SII (bold). To simulate the data from Sagar and Rakowski (1994), additional assumptions about cytoplasmic concentrations and the free energy of ATP must be made. For all simulations, the cytoplasmic values for ATP, ADP, and P\textsubscript{i} can be found in Table I of the main article text. Sodium and potassium values can be found in Fig. S4.

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Given an initial set of model parameters, the steady state current as a function of membrane potential was computed for each of the extracellular potassium concentrations examined by Sagar and Rakowski (1994). For each potassium concentration, the membrane potential was discretized and the reaction scheme simulated to determine the steady state current. This family of curves was then scaled by a common parameter to find the best simultaneous fit to all of the experimental data. Such a scale factor is required to normalize the single pump current to the experimentally measured current, which is the result of many pumps. This root mean square fitness score was then re-
Figure S4. Na-K-ATPase reaction scheme and current voltage fit. (A) This reaction scheme combines aspects of two previous models published by Heyse et al. (1994) and Sokolov et al. (1998). The basic tenants of this model follow the accepted Post-Albers description that involves the pump switching between an inward and outward facing conformation, E1 and E2. Four cartoon diagrams have been included to illustrate the physical state of the enzyme. Chemical states within grayed regions are assumed to interconvert on a very fast time scale and are mathematically treated as one state. Please refer to the original papers for details concerning the construction of the differential equations from this reaction scheme. (B) The kinetic scheme in A is simulated using the model parameters in Table SII. The total number of pumps giving rise to the measured current is unknown; thus, an arbitrary scale factor has been used to scale the single pump current (solid line) to the experimentally measured data (○). Cytoplasmic sodium and potassium concentrations are held constant at 20 and 120 mM, respectively. Lumenal sodium concentration is initially 120 mM. Data is adapted from Figure 5 A of Sagar and Rakowski (1994).

<table>
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<th>Parameter</th>
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Parameters used for the Na-K-ATPase pumping profile. *Parameters differ from the values presented in the references. The kinetic scheme presented in Fig. S4 A has four closed loops. Detailed balance has been enforced around each of these loops by constraining the reverse rates of the parameters set off in bold. The remaining four * parameters correspond to reverse rates along the potassium pumping portion of the cycle. These parameters have been determined from fitting current voltage data (Fig. S4 B).
corded. A Nelder-Mead algorithm was used to search through the space of model parameters and find the set that minimized the fitness score (Press, 1997). These parameters are presented in Table SII, and the fit to the data is presented in Figure S4 B. In all figures, ionic flux from the cytoplasm to the extra-cellular/lumenal space is defined to be positive, and the membrane potential is defined relative to the lumen. Thus, a positive membrane potential will induce positive charges to flow in the negative direction.

This calibrated model has been used to compute the sodium and potassium rates over a wide range of external/lumenal concentrations and membrane potentials. To illustrate what a slice of this surface looks like, the sodium rate is plotted as a function of membrane potential and lumenal potassium concentration in Figure 2B. It is this response surface combined with the corresponding surface for the potassium flux that are used to simulate the effect of incorporating Na-K-ATPases in model vesicles. When creating these surfaces, cytoplasmic concentrations of all species are assumed constant. Values can be found in Table I of the main article text.

These computations are given in program files: nak_pump.m (main program); delta.m; nak_param.m.

Buffering Capacity

When a proton enters the lumen or the cytoplasm, it will equilibrate with the lumenal matrix. A bound proton is then no longer free to contribute to the pH in that compartment. The ability of the matrix to bind free protons is referred to as its buffering power. The standard definition of buffering power originally given by Michaelis is (Roos and Boron, 1981):

$$\beta = \frac{\Delta [B]}{\Delta pH}$$  \hspace{1cm} (1)

where $\Delta [B]$ is the amount of strong base added to the system. Since $\Delta [B] \equiv \Delta [H^+]$, the relation between change in pH and change in proton concentration is determined. Some experiments suggest that the buffering capacity of some organelles is nearly constant in the pH range 6–8 (Farinas and Verkman, 1999; Wu et al., 2000). However, Van Dyke and Belcher (1994) find that the buffering capacity of endosomes depends strongly upon the luminal pH (Fig. S5).

In the simplest case, there is only one kind of proton accepting site with some intrinsic $pK_a$. Once the internal pH goes below this $pK_a$, the buffering ability of the matrix quickly diminishes. From the definition of pH, one then obtains a buffering capacity, $\beta \propto 10^{-pH}$. In reality, the matrix is composed of many titratable sites with a distribution of $pK_a$'s. With this interpretation, the jagged variation in the buffer capacity found by Van Dyke and Belcher (1994) seems more reasonable.

A sensitivity analysis shows that the buffering capacity is one of the most dominant factors in determining the shape of the acidification curves. Originally, it was our goal to try to recreate the pumping performance of the V-ATPase from the experimentally measured curves in Figs. 3 and 4 (text). This would serve as a calibration for the proton pump. However, uncertainties in the volume and buffering capacity made it impossible to extract dependable information about the pump.
Surface Potentials

Here we address how our description of pH regulation is affected by negatively charged lipids on the cytoplasmic leaflet of organelles. First consider an organelle permeable to ions, but with no surface charge or pumps. At equilibrium, only a Donnan potential, $\Delta \psi$, will exist across the organelle due to the impermeable macro-ions. If a negative surface potential is imposed on the organelle, $-\Delta \phi$, the surface values of positive (negative) ions in solution will increase (decrease) by a Boltzmann factor (see Eq. 4 in text). The concomitant flow of ions quickly builds up an opposing membrane potential, $\Delta \psi \to \Delta \psi + \Delta \phi$. Changes in luminal concentrations are marginal since very few ions are required to produce a sizable membrane potential. Thus, the addition of a surface potential shifts the balance of the (equilibrium) electrochemical potential from a chemical to an electrical gradient, while the magnitude remains unchanged. However, steady state and transient dynamics do depend upon surface potentials since channel and pump activities depend on the relative magnitudes of the concentration and electrical components of the electrochemical potential.

Consider the determination of Golgi proton permeability in Fig. 6, inset (text). In the absence of surface charge, membrane potentials are very small and an analytic analysis is possible. To see how this simulation depends on surface potential, we must consider our model for proton flux:

$$ J_{p=0} = P \cdot S \cdot U \cdot \frac{([H^+]_L - [H^+]_C \cdot e^{-U})}{1 - e^{-U}} = P \cdot S \cdot ([H^+]_L - [H^+]_C \cdot e^{-U}). \quad (2) $$

When the reduced surface potential, $-\Delta \phi$, is added $[H^+]_C \to [H^+]_C \cdot \exp(+\Delta \phi)$ and $U \to U + \Delta \phi$, as discussed above. Because of the exponential Boltzmann factor, the membrane potential is no longer negligible. The proton flux is:

$$ J_{\Delta \phi \neq 0} = P \cdot S \cdot (U + \Delta \phi) \cdot \frac{([H^+]_L - [H^+]_C \cdot e^{(U + \Delta \phi)})}{1 - e^{(U + \Delta \phi)}} $$

$$ = P \cdot S \cdot \Delta \phi \cdot \frac{([H^+]_L - [H^+]_C \cdot e^{U})}{1 - e^{\Delta \phi}} $$

$$ = J_{\Delta \phi = 0} \left( \frac{\Delta \phi}{1 - e^{-\Delta \phi}} \right). \quad (3) $$

where the approximation $U + \Delta \phi \sim \Delta \phi$ has been made. When $\Delta \phi = -50 \text{ mV}$, the cofactor in the last equation =2.27. When the simulation is rerun without the surface potential, the best fit permeability is exactly 2.27× larger than the one reported in Table IV (main article text).

From this example, we see that proton fluxes can be two to three times smaller in the absence of surface charge. This makes the pH simulations in Figs. 3 and 4 (text) undershoot the present trajectories by $\sim$0.2–0.3 pH units, if the proton pump remains unaffected by the changes in membrane potential and surface concentrations. However, in our model for the proton pump, the rate-limiting step near cytoplasmic pH values is the capture of a proton from the cytoplasm. Removal of the $-50\text{-mV}$ surface potential lowers the surface concentration of protons by a factor of 10. This decreases the pumping rate by $\sim$1.7× when the lumen is near neutral pH. Examination of the sodium performance surface (Fig. 2 B, text) in the absence of a surface potential reveals a less dramatic influence on the dynamics ($\sim$6 Na$^+$/s). Thus, only the exact pump numbers and permeability values change in the absence of surface potentials, and the nature of our presentation is unaffected.

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