THE VISIBILITY OF SINGLE LINES AT VARIOUS ILLUMINATIONS AND THE RETINAL BASIS OF VISUAL RESOLUTION

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I

Nature of Problem

The present measurements were made to explore the problem of why visual acuity varies with illumination. This question is not limited to the human eye, nor to the vertebrate eye (cf. Ehrenhardt, 1937) but is a general one since even insects (Hecht and Wolf, 1929; Hecht and Wald, 1934) and crabs (Clark, 1935) show a similar dependence of visual acuity on illumination. The matter has been extensively studied in man (Uhthoff, 1886; Koenig, 1897; Hecht, 1928) merely because the measurements are conveniently made and the results are of common interest. The data for all organisms show that the capacity for resolving visual detail increases in a definite way with the intensity of illumination.

A quantitative explanation of this phenomenon was suggested several years ago (Hecht, 1928; Hecht and Wald, 1934), and is based on the supposition that the receptor elements in the eye vary in sensibility over a wide range of light intensities, so that their thresholds are distributed in the usual manner of populations. This statistical distribution need not be a fixed property of the individual elements, but may be variable in time (Hecht and Wolf, 1929) depending perhaps on the recovery of the individual elements following stimulation as suggested by Berger and Buchthal (1938 b). In terms of this distribution the number of elements functional in a given retinal area increases with the illumination, and since the resolving power of a receiving surface varies with the number of active elements per unit...
area, it follows that the resolving power or visual acuity of the eye should increase as the retinal illumination increases.

This explanation requires no knowledge of the distribution of light in the image formed on the retina, and hence does not concern itself with the actual nature of visual resolution. However, the appearance of the image on the retina in terms of diffraction optics is certainly an important factor in visual resolution (Hartridge, 1922; Shlaer, 1937). We therefore thought that a detailed study of it may point the way toward a formulation of the relation between visual acuity and illumination in terms which are more concrete than the statistical one already proposed.

The ordinary test objects used in visual acuity measurements are unit configurations like letters, hooks, or broken circles. These are complicated patterns and a calculation of the exact light distribution in their retinal images is almost impossible. Therefore, it is difficult to determine precisely what these test objects measure (cf. Berger, 1936; Berger and Buchthal, 1938 b). A simpler test object is an evenly spaced grating. However, because this is a repeating pattern, limits to its resolution may be set by factors other than intensity distribution (Abbe, 1873; Shlaer, 1937). The simplest unit test object is a single opaque line against a uniformly illuminated background, and the light distribution produced by it on the retina may be determined in complete detail in terms of diffraction optics (Rayleigh, 1903; Hartridge, 1922). We therefore measured the relation between brightness and visual resolution using such a test object, in order to describe as nearly as possible what happens at the retina.

II

Method and Apparatus

The apparatus is shown diagrammatically in side view in Fig. 1. A large sheet of flashed opal glass G, selected for uniformity, is illuminated by a short focus lantern-slide projector LCP, to furnish a surface of uniform brightness. The opal plate is outlined as a disc 2 feet in diameter by an opaque shield S, and serves as the background for the test object. The test objects are wires W of different thickness; the finer wires are of drawn tungsten, while the coarser are brass rods. Each wire is separately mounted across the diameter of a brass ring R, which is slightly larger than the illuminated background. Three hooks H in the
opaque shield hold the brass ring in place in front of the illuminated background; the brass ring may be rotated in its own plane, so that the wire occupies any selected direction.

The brightness and color of the illuminated background is varied by a series of neutral and color filters \( F \) placed immediately in front of the projection lens \( P \). The maximum brightness is just above 30 millilamberts. With the proper Wratten filters the brightness may be decreased in steps of almost any size down to any value desired. A heat absorbing glass \( A \) is placed between the condensers in order to protect the gelatin filters from the heat of the projection system; this renders the light slightly greenish.

![Diagram of the optical arrangements for visual resolution of a single line against a uniformly illuminated background.](image)

**Fig. 1.** Diagram of the optical arrangements for visual resolution of a single line against a uniformly illuminated background. \( LCP \) is a lantern slide projector with its lamp \( L \), condensers \( C \), projection lens \( P \), and heat absorbing glass \( A \). Neutral filters \( F \) control the illumination on the flashed opal glass \( G \). The opaque shield \( S \) outlines the field in front of which is the wire \( W \) set in the ring \( R \) and held by the hooks \( H \); the sketch at the right shows how these are arranged from the observer’s point of view.

The lamp and projector are in an anteroom, with the projection lens \( P \) just protruding through an opening into a small dark room. The opal glass and opaque shield are in the doorway between this small dark room and another larger dark room in which the observer is. The part of the apparatus facing the observer is shown at the right in Fig. 1. When the projection lamp is on, the observer does not see the brass ring; he sees only the illuminated circular surface and the wire in front of it. The observer sits in a chair on rollers, which moves over the floor near a scale so arranged that the position of his eyes at any moment may be read from a pointer attached to the chair and in contact with the scale on the floor.

Before making measurements, the observer stays in the dark for about 20 minutes. He is then presented with the lowest illumination to be investigated,
in front of which the wire has been placed in one of three specific positions by the recorder. The observer adapts to this illumination for a few minutes, after which he moves towards the wire until he can describe its direction with certainty. All observations are binocular, with the natural pupil. Three settings are usually made with each intensity, and the observer always adapts for a few minutes to each intensity before beginning a determination. The procedure is continued until the whole intensity range is studied, and usually takes about an hour and a half.

### TABLE I

**Relation between Background Light Intensity and the Visual Angle Subtended by an Opaque Line When It Just Becomes Visible**

<table>
<thead>
<tr>
<th>Light intensity in millilamberts</th>
<th>Visual angle in minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Series 1</td>
</tr>
<tr>
<td>0.00000447</td>
<td>16.14</td>
</tr>
<tr>
<td>0.00000603</td>
<td>4.94</td>
</tr>
<tr>
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<td>1.50</td>
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<tr>
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<tr>
<td>0.00302</td>
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<tr>
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</tr>
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<td>0.0867</td>
</tr>
<tr>
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<tr>
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<tr>
<td>0.661</td>
<td>0.0154</td>
</tr>
<tr>
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<td>0.0130</td>
</tr>
<tr>
<td>6.92</td>
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</tr>
<tr>
<td>30.2</td>
<td></td>
</tr>
</tbody>
</table>

The influences of the distance from the test object (Freeman, 1932) and of the size of surround (Lythgoe, 1932; Hecht and Smith, 1936) are eliminated by keeping the final resolution distance of the observer between 2 and 3 meters from the test object. This is done by using wires between 12 μ and 8 mm. in thickness.

### III

**Measurements**

We made three series of measurements, all with S. H. as observer and E. U. M. as recorder. Series I was exploratory, and involved
three separate runs; we used such wires as were easily available and determined the general relation between angular size and brightness. After this, some of the wires were changed and the light source was put on alternating current and its voltage kept constant. We then made series II which also consisted of three independent runs. Finally, several weeks later, after further changes in the wires, and

![Diagram](image_url)

**Fig. 2.** The relation between the brightness of the background and the visual angle subtended by the thickness of the wire when it just becomes resolved against the illuminated background. Series 1, 2, and 3 are shown separately; the points for series 3 are in their correct positions on the coordinates while series 2 and 1 are moved to the right 1 and 2 log units respectively. Note the separate relations shown by the low intensity measurements and the high intensity ones, corresponding to rod and cone functions respectively. The six curves drawn through the data represent the theoretical equation $\alpha = b(1 + (1/KI)^{1/2})$ in which $\alpha$ is the visual angle, $I$ the light intensity, and $b$ and $K$ are two constants.

a minor change in the filter arrangements we made the third series of measurements consisting as before of three separate runs. The averaged measurements for the three series are given in Table I. It is apparent from Table I that both the light intensity and the just resolvable visual angle corresponding to it vary over a large range; for convenience of examination, therefore, the data are plotted in Fig. 2 on a double logarithmic grid.
Perhaps the most obvious property of the measurements as shown in Fig. 2 is their natural separation into two sections, one at the lower brightnesses, and the other at the higher brightnesses. By now this is a familiar phenomenon, and a similar division has been found in almost all visual functions when they have been measured over a wide intensity range (Hecht, 1937a). It is practically certain that the low intensity section represents the behavior of the rod system in the retina, while the section at high intensities represents the behavior of the cone system, in accordance with the duplicity theory first suggested by Schultze (1866) and later developed by von Kries (1929) and Parinaud (1881).

In this connection we made some preliminary measurements using violet light instead of white. In terms of the relative spectral sensitivities of the rods and the cones, it is to be expected that with violet light the separation of the two segments of the data will be greater than with white light. This is true for dark adaptation, flicker, intensity discrimination, visual acuity, and instantaneous threshold (cf. the review by Hecht, 1937a). Our measurements, though too few to be recorded here, show the same phenomenon. The low intensity section for violet is about 1 log unit to the left as compared to that for white light, while the high intensity sections are, of course, in the same position for both. We may therefore safely conclude that the two sections of the data in Fig. 2 correspond to the histological and functional duality of the vertebrate eye as expressed by the duplicity theory.

It is to be noted in Fig. 2 that the fifth point from the right is above the curve in each of the three series. This is true for each run made, and is a real phenomenon. We measured this point with a variety of combinations of wires, distances, and filters, but it always came out slightly to one side of the line, as if at this intensity there is some change in retinal function. However, the point is not correlated with any subjective phenomenon, and its meaning is obscure.
One of the striking things which the measurements show is the great range of resolution of which the eye is capable. At the lowest light intensities the eye can just see a line whose thickness subtends a visual angle of about 10 minutes, while at the highest intensity the just resolvable line subtends only 0.008 minute which is very nearly 0.5 second of arc. This is a range of about 1 to 1200, and is 10 or 20 times the range ordinarily found with test objects like a broken circle, a hook, or even a grating. With such objects the resolution at the lowest light intensities is usually between 20 and 30 minutes of arc, while at the highest intensities it is between 0.5 and 1 minute, thus covering a maximum range of only 1 to 60.

This difference in range shown by the two kinds of test object occurs mainly in the upper light intensities. At low brightnesses, the two yield values of the same order of magnitude; at high intensities the resolution with single lines attains a value about 60 times as great as with a hook or with a grating.

It is worth noting that these differences in the resolving power of the retina between a single line and a grating are not confined to the human eye. Recently Ehrenhardt (1937) has measured with a lizard the relation between the intensity of illumination and the visual acuity. With a grating made of equally wide spaces and bars the resolution at the lowest intensity corresponds to a visual angle of very nearly 1 degree, while at the highest intensities it corresponds to an angle of 11.5 minutes; this is a range of about 1:5. However, with a grating made of spaces 30 times wider than the bars, the resolution at the lowest light intensities is about 30 minutes, while at the highest intensities it corresponds to about 1.3 minutes, making a range of 1:25. Note here too that at low light intensities the just resolvable visual angle is much the same for both types of test object, and that the difference in range is the result of the better resolving power for widely spaced lines at the high light intensities.

A similar situation holds for the bee's eye. With a grating of equal bars and spaces, the highest resolution corresponds to an angle
just equal to the smallest ommatidial separation (Hecht and Wolf, 1929). On the other hand, a single bar can be resolved even when it subtends an angle only a quarter as wide as an ommatidium (Buddenbrock, 1935).

VI

Maximum Resolution and Intensity Discrimination

The values obtained at the highest intensities deserve closer examination. At a maximum brightness of 30 millilamberts we could resolve with certainty a wire which subtends only 0.5 second of arc. However, Fig. 2 shows clearly that a slightly smaller angle would probably be resolved at still higher brightnesses. As it is, 0.5 second is a much smaller angle than the minimum of 3 to 4 seconds found by Hartridge (1922) for single lines. A number of individuals in the laboratory confirmed this small angle, most people having no difficulty in achieving it (cf. also Shlaer, 1937). We attribute this to the evenness of the illuminated background; the just resolvable angle increases rapidly when the background is irregularly illuminated. For this reason we gave up very early the efforts to make the measurements with the open sky as background.

The geometric image on the retina of the wire corresponding to the highest resolution is barely 0.04 μ wide. Since the central cones of the fovea are between 2.0 and 2.6 μ in diameter (Rochon-Duvigneaud, 1907; Østerberg, 1935), the geometrical image is only about 1/60 of the width of a single cone.

However, as Rayleigh (1903) and Hartridge (1922) have pointed out, such computations of retinal distances are meaningless, since the image on the retina is too small to be described in the simple terms of geometrical optics. Because of diffraction at the pupil, the image of a wire is not a sharp black shadow covering only a small fraction of a cone, but a fine fuzz of a shadow extending over several cones. The distribution of light under such conditions has been carefully worked out by Rayleigh (1903), and though tedious to compute, is straightforward and simple.

Measurements of the pupil size, made during the course of our work showed that the pupil never was smaller than 3 mm. in diameter even at the highest illuminations. We have therefore used this
value in the computation of light distribution on the retina in terms of diffraction. It is significant in this connection that visual acuity, though it varies sharply with pupil diameter below 3 mm., is practically constant for pupils larger than 3 mm. (Lister, 1913; Cobb, 1915; Berger and Buchthal, 1938a). The constancy of visual acuity for pupils larger than 3 mm. is probably due to the neutralization of two tendencies of the light distribution in the retinal image: as the result of diffraction the image becomes better as the pupil increases, while as the result of chromatic aberration it becomes worse as the pupil increases (Hartridge, 1918; 1922).

![Diagram of light intensity distribution](image)

Fig. 3. Light intensity distribution in the diffracted retinal images produced by three wires whose geometrical images are shown as three lines in the lower part of the figure. The pupil is taken as 3 mm. The scale on the top of the figure represents the retinal mosaic, consisting of cones whose diameters are 2.3 μ, this being the average size of the group of cones in the very center of the fovea.

We have computed the light distribution in the retinal image of a series of wires, using Rayleigh's equations and a pupil of 3 mm. As an example, Fig. 3 shows the results for three lines whose geometrical images are 0.3, 0.5, and 2.6 μ wide, corresponding to visual angles of 4.1, 6.8, and 35.5 seconds respectively. The distribution corresponding to the smallest resolvable angle (0.5 second) is so flat and near the upper edge of the ordinates that it can hardly be shown in a drawing of this scale. However, it has the same form as the others, and its vertical dimensions are a constant fraction of those of the larger distributions. Indeed, this is true of nearly all the sizes, except only the largest wires. The curves in Fig. 3 give the light intensity
at any point in the retinal image; the region below any curve means light intensity while the region above any curve means shadow or absence of light. The scale at the top is marked off in units of 2.3 μ representing the average cone separation in the fovea. By determining the area of light under the curve for each cone one can compute the relative amount of light falling on each cone.

Hartridge (1922) first made a computation of this sort for a wire subtending 8 seconds of arc. He used a pupil of 2 mm. and a cone width of 3.2 μ, and found a difference of 10 per cent between the light on the central cone and the ones on either side. He chose an angle of 8 seconds for computation because of Rayleigh's theoretical expectation that this is the minimum resolvable. However, he actually found a value of about between 3 and 4 seconds for the resolution of a black line against an illuminated white surface, which would make the difference in light on two adjacent rows of cones more nearly 5 per cent. Even so, this is a fairly coarse intensity difference, and it is hard to see how it can be the limiting factor in visual acuity. One cannot explain this by recalling that intensity discrimination becomes poorer with small areas (Lasareff, 1911; Heinz and Lippay, 1928; Steinhardt, 1936), because small areas are not involved in such measurements. To be resolved at these fine angles, a line must be long, a great many times longer than it is wide; in other words, the number of retinal elements involved is large, and corresponds to areas such as yield the best values for intensity discrimination.

The present situation is indeed different from that envisaged by Hartridge. The best modern histological measurements (Rochon-Duvigneaud, 1907; Østerberg, 1935) show the central cones of the human retina to be between 2.0 and 2.6 μ. Moreover, our best resolution with a 3 mm. pupil corresponds to 0.5 second. When the computation for intensity distribution in such a retinal image is carried through for these data, we find a much more critical condition. Assuming the general illumination on the retina to be 100 per cent, then it comes out that a central row of cones 2.3 μ wide is illuminated by 98.83 per cent of the prevailing intensity, while the row to either side has a light intensity of 99.78 per cent. The difference in light intensity between the two rows is 0.95 per cent, and is a value near those usually found in measurements of intensity discrimination at
high light intensities. It corresponds to a value of $\Delta I/I = 1/105$, and is exceeded only by Aubert's (1865) best value of $1/146$ and Helmholtz's (1866) value of $1/167$ at the highest illuminations. For more moderate illuminations like our maximum the values generally range around $1/100$.

This computation tells us that a fine line is recognized at such small angles because even its fuzzy and extended shadow reduces the light on one long row of cones to a value which is just perceptibly less than the light on the row of cones on either side of it. The line appears sharp because it produces a perceptible shadow on one row of cones only. Note that the row of cones to either side of this critical row has its illumination reduced by only 0.22 per cent compared to the prevailing illumination next to it, and that this small difference in intensity cannot be perceived; it would mean a $\Delta I/I$ of about $1/400$ and cannot be achieved by the eye. Thus when a line becomes recognizable, the light distribution which it produces on the retina is such that only one row of cones is just perceptibly shaded by it.

From Hartridge's original computation it was not clear why a line is perceived as a sharp line instead of as a band of shadow gradually fading at the edges; and it has been necessary to assume some central nervous mechanism for converting such a gradual distribution into a sharp line. However, in view of our present computation the line appears sharp simply because its diffracted image on the retina affects only one row of cones differently from the rest. Thus Hartridge's idea that the maximum resolving power of the eye is determined by its capacity for intensity discrimination is even better than originally anticipated.

We have supposed that in the retina the comparison is made between the row of perceptibly shaded cones and the row immediately next to it. This makes an intensity difference of just 0.95 per cent between the two rows. However, in view of the nature of intensity difference perception (Hecht, 1935), it is even more likely that what is recognized is the intensity difference produced on the central row of cones from the time it is affected by the general retinal illumination to the time when it is affected by the shaded center of the image, and these successive intensities are achieved by the obvious eye movements made during the effort to resolve the line in the field. Considered
this way, the intensity difference is about 1.17 per cent, which is also near the minimum perceptible intensity difference, but which leaves room for perception of the still smaller differences which must correspond to the better resolution shown by Fig. 2 as possible at higher illuminations than those we used.

VII

Visual Resolution at Different Illuminations

If the highest resolving power of the eye is determined by its maximum capacity for intensity discrimination in the diffracted retinal image, is it not possible that the resolving power of the eye at any illumination is also determined by the intensity discrimination of the retina at that illumination? One may put this more directly. The essential point of the explanation of maximal visual resolution is that because of the transformation of the optical image by diffraction, the limiting factor in detail recognition is the capacity of the eye for intensity discrimination. It is well known that the capacity for intensity discrimination varies with the light intensity; therefore the visual resolution of detail must also vary with the light intensity.

The distribution of light in the diffracted image of a single line depends on the pupil size and on the width of the line, but not on the intensity of the light. Therefore with a fixed pupil, the distribution depends on the width of the line, which at once sets the depth of shadow in the diffracted image, and fixes the light intensity on the central row of cones as compared either with that prevailing generally on the retina or with that on the row of cones to either side of the central row. The fractional difference in light intensity produced on the retina is thus a fixed property of a given size of line, and has a specific value \( \Delta I/I \), where \( I \) is the background intensity and \( \Delta I \) is the difference between it and the intensity on the shaded central row of cones. Since the just perceptible fractional intensity difference \( \Delta I/I \) has a different value for each light intensity (Aubert, 1865; Koenig and Brodhun, 1889; Hecht, 1935), the line will be resolved only at intensities which are equal to or greater than the value of \( I \) for which the particular fraction \( \Delta I/I \) is the just perceptible one. This is for a fixed size of line and a variable intensity.
A similar situation exists for a fixed light intensity $I$ and a test object of variable size. The fixed value of $I$ has, corresponding to it, a minimum fractional intensity difference which the retina can just recognize. If now the opaque line produces an intensity distribution in its image on the retina which results in a fractional intensity difference on the central row of cones equal to or greater than the critical $\Delta I/I$ for this value of $I$, then the line will be resolved; if the fractional intensity difference is less than the critical $\Delta I/I$ then the line will not be resolved. A measurement will then consist in finding the correct angular size for the test object so that the intensity distribution on the retina will just produce the critically perceptible value of $\Delta I/I$ for that intensity.

In order to render this explanation quantitative it is necessary to discover the precise way in which the visual angle subtended by the wire test object is related to the fractional intensity difference it produces on the retina. In first approximation this may be done quite simply. Fig. 3 shows that the retinal shadow produced by a line extends over about 5 or 6 cones. Because the geometrical image of the wires in the high intensity cone section of the data is in the main only a fraction of a cone in width, the form of the intensity distribution in this shadow is practically constant, and will vary only in vertical dimension in direct proportion to the width of the geometrical image of the wire. Thus in Fig. 3 it is not possible to distinguish any difference except in vertical depth between the shape and extent of shadows produced by geometrical images 0.5 $\mu$ and 0.04 $\mu$ wide. In these cases, the middle three rows of cones receive about 98 per cent of the total amount of the shadow, while the central row of cones alone receives about 70 per cent of the shadow. Thus as a first approximation the main difference produced by changing the diameter of the wire is in the actual density of the shadow falling on the row of cones occupying the center of the shadow. But the vertical density of the shadow on this central row of cones corresponds to the fractional difference in light intensity $\Delta I/I$ between the general background illumination $I$ and that on the central row of cones $I - \Delta I$, and is indeed directly proportional to it. In other words, the fractional intensity difference $\Delta I/I$ produced on the retina by these fine wires is, as a first approximation, directly proportional to the
visual angle they subtend. Therefore, if the limiting factor in visual resolution is intensity discrimination, then the relation between the just resolvable visual angle and the light intensity should be the same as the relation between the just discriminable fraction $\Delta I/I$ and the light intensity.

The essential condition for this derivation is that while the diffracted image of the wire extends over 5 or more cones, its geometrical image covers only a fraction of a cone. This condition holds over nearly the whole of the range of visual angles resolved by the cones. It begins to break down at the lowest cone resolutions because the geometrical images here begin to be wider than one cone. But at these low intensities the function is taken over by the rods, and the situation becomes quite different.

It is known anatomically for the central, foveal cones that they have a one to one connection with optic nerve fibers. This is borne out by the maximal resolution of gratings which corresponds precisely to the dimensions of single cones (Shlaer, 1937). For the rods, however, it is established that many elements are connected with a single nerve fiber, and this corresponds to their very low resolving power. In other words the unit of receptor action cannot be a single rod, but must be many rods. How many, one cannot say, but judging by the relative numbers of rods and nerve fibers it is certainly larger than ten, and may be a hundred or even more, though there is no reason to suppose the number of rods in a receptor unit to be the same for the whole retina. For our present purpose the precise number does not matter so long as it is more than a few rods. For convenience assume the number as five in linear dimension. Since the rods are about the same size as the central cones (Østerberg, 1935), the whole of the diffracted image of the wire will fall on this rod unit, and will be recognized only as a total decrease in the light. Any change in the depth and width of the shadow produced by the diffracted image of the wire results only in a decrease in the total light falling on the rod receptor unit; and this decrease will be directly proportional to the angular width of the wire since this determines the total amount of light removed. In short, here too the fractional intensity difference $\Delta I/I$ will be directly proportional to the angular width of the wire. As for the cones, so for the rods: provided that the
limiting factor in resolution is the retinal intensity discrimination, then the relation between visual angle and intensity should be the same as the relation between $\Delta I/I$ and intensity.

It is fortunate that the relation between the light intensity and the just perceptible fractional difference in intensity is known not only for the human eye, but for several other organisms as well. Moreover all the critical data for all animals have the same form, and can be described with high precision by the same type of equation (see the review by Hecht, 1937a). For the human eye the equation is

$$\Delta I/I = c[1 + 1/(KI)]^{1/2} \tag{1}$$

where $I$ is the prevailing light intensity, $\Delta I$ is the just perceptible increment or decrement in it, and $c$ and $K$ are two constants of which $c$ is the minimum value of $\Delta I/I$ obtained at the highest intensities, and $K$ is the reciprocal of the intensity at which $\Delta I/I$ has a value of 4 times the minimum. The two constants have different values for cone vision and for rod vision, but the form of the equation is the same for both.

We have just seen that as a first approximation the critical visual angle $\alpha$ for the resolution of a wire is directly proportional to $5I/I$, that is $\alpha = b'\Delta I/I$. When this value is substituted in equation (1) it yields

$$\alpha = b[1 + 1/(KI)]^{1/2} \tag{2}$$

where $b = cb'$. If equation (2) is plotted as log $\alpha$ against log $I$, its form is invariant and independent of the numerical values of $b$ and $K$. It is this equation (2) which is actually drawn through all the data in Fig. 2 both for the rod sections and for the cone sections. The value of $b$ merely determines the position of the curve on the ordinates, while that of $K$ fixes it on the abscissas. It is apparent that the equation describes both parts of the data with reasonable fidelity, and this may be taken as evidence for the idea that at any intensity the visual resolution of a single line is determined by the power for intensity discrimination of the retina at that light intensity.\(^1\)

\(^{1}\) In comparing measurements of visual function over a wide range of light intensities with such theoretical curves as given by equation (2) it is necessary to correct for varying pupil diameter (Reeves, 1918) and for pupil efficiency (Stiles
Our original purpose in using a single line was to enable us to describe in almost complete detail the light distribution on the retina. This has resulted in the present generalization about the interrelation between intensity discrimination and visual resolution at all intensities. May it not be that the same interrelation holds not only for the single line but for more complicated test objects as well?

It is certain that for at least one test object the interrelation cannot be carried over in its direct form. This is for the case of a repeating pattern such as a grating, because many factors besides intensity discrimination enter into the measurements. For instance, the maximum grating resolution can be sharply set by the dimensions of the receptor mosaic. This is true for the bee's eye (Hecht and Wolf, 1929); it also holds for the human eye (Shlaer, 1937) even when the intensity differences on the retina are quite large. Moreover in the human eye, pupil size can limit grating resolution not as it affects diffraction and intensity distribution but as it limits the transmission of the diffraction spectrum produced by the grating (Abbe, 1873; Shlaer, 1937). In other words, a grating, by its nature as a repeated pattern introduces extraneous elements into the problem.

Other test objects like a broken circle or a hook, though more complicated than one line, are nevertheless like it in being single units, and it is hard to see a really basic difference between such test objects and a line. The broken circle or the hook must suffer the same transformation of geometrical image as the wire; it is merely that the exact computation of the intensity distribution becomes very difficult indeed, so that one cannot envisage its precise relation to the retinal mosaic.

and Crawford, 1933). Examples are Blanchard's data of instantaneous threshold (Hecht, 1937 b) and Koenig's data of visual acuity (Hecht, 1937 a). We have made such corrections in the present data, and find that because of the comparatively narrow range of intensities covered by each segment of the data, the pupil corrections do not change significantly the relation of the points in each segment to one another. As a result the theoretical curve fits the data either corrected or uncorrected for pupil.
A significant point in this connection is that when the relation between visual acuity and light intensity is measured even with the common single test objects, the data also follow equation (2). This is true of Koenig's (1897) classical data using a hook as test object, which have been recomputed and replotted (Hecht, 1937a) and of Shlaer's (1937) very refined and critical data using a broken circle. Note that visual acuity is generally defined as the reciprocal of the just resolvable angle \( \alpha \) in minutes of arc subtended by the critical element in the test object, and that equation (2) may be plotted in its stationary state form as done by Shlaer. This agreement of visual acuity data with intensity discrimination data cannot obviously be considered as proof of their identity; the agreement, however, is pointed, when considered in the light of theoretical expectation.

**SUMMARY**

The visual resolution of a single opaque line against an evenly illuminated background has been studied over a large range of background brightness. It was found that the visual angle occupied by the thickness of the line when it is just resolved varies from about 10 minutes at the lowest illuminations to 0.5 second at the highest illuminations, a range of 1200 to 1.

The relation between background brightness and just resolvable visual angle shows two sections similar to those found in other visual functions; the data at low light intensities represent rod vision while those at the higher intensities represent cone vision. With violet light instead of white the two sections become even more clearly defined and separated.

The retinal image produced by the finest perceptible line at the highest brightness is not a sharp narrow shadow, but a thin broad shadow whose density distribution is described in terms of diffraction optics. The line of foveal cones occupying the center of this shadow suffers a decrease in the light intensity by very nearly 1 per cent in comparison either with the general retinal illumination or with that on the row of cones to either side of the central row. Since this percentage difference is near the limit of intensity discrimination by the retina, its retinal recognition is probably the limiting factor in the visual resolution of the line.
The resolution of a line at any light intensity may also be limited by the just recognizable intensity difference, because this percentage difference varies with the prevailing light intensity. As evidence for this it is found that the just resolvable visual angle varies with the light intensity in the same way that the power of intensity discrimination of the eye varies with light intensity.

It is possible that visual resolution of test objects like hooks and broken circles is determined by the recognition of intensity differences in their diffracted images, since the way in which their resolution varies with the light intensity is similar to the relation between intensity discrimination and light intensity.

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