ENERGY AND VISION.

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Although a large number of papers and books have been published on the problems of vision (1), a very limited amount of work is to be found on the minimum energy necessary to produce visual sensation. The classical work of Langley (2) for different wave lengths the papers of Grijns and Noyons (3), Zwaardemaker (4), and Kries (5), for white light are always quoted, but the figures given by different authors do not always agree, discrepancies of 100 per cent, sometimes of 1000 per cent, being frequent, with no explanation. For this reason, it was thought advisable to check all these figures, in order to ascertain whence came the discrepancies. Furthermore, as Langley's figures are given by himself with a certain degree of approximation, and were calculated for the light emitted by the sun, we thought it would be interesting to check them by another method, for another source of light, the Nernst lamp, for instance. These are the reasons for carrying on this series of measurements.

In order to give an idea of how difficult it is to find a figure corresponding to the minimum visible for a certain wave length, we will give an example. Langley's figures are quoted as follows for the wave length 0.55μ, for which the human eye shows a maximum of sensitivity:

By Broca (6) .................................... 5.6 \times 10^{-9} \text{ ergs}
By Henri and des Bancels (7) ...................... 3.0 \times 10^{-8} \text{ ergs}

whereas Langley's real figure, as given in his paper, is 2.8 \times 10^{-9} \text{ ergs}. Furthermore, Henri and des Bancels state on another page that 10^{-10} is the order of magnitude of the minimum energy necessary to produce the sensation of vision in the green (0.55μ), and Langley (2)¹ states that it is 1.0 \times 10^{-8} (for practically the same radiation, 0.53μ). In order to clear this matter up, we have to go over Langley's paper carefully. Langley states and gives a solution for two different problems: first, determination of the intensity of light necessary to read a table

¹ Langley, (2), p. 23.
of logarithms or to discern any arbitrary character and second, determination of the minimum visible; namely, the minimum of energy which can produce the sensation of light on the retina. The results of the first determinations are expressed in tables (2) in function of the wave length and of the sensitiveness of the eye, in arbitrary figures related to the apparatus and inversely proportional to the energy. The results of the second determinations are expressed, on the contrary (2) in the following way: reciprocals of calories = reciprocals of ergs (let us call this Table A), and these values stated in terms of horse-power (Table B). Now the meaning of these tables is quite ambiguous, and it is not surprising that authors have been mistaken in quoting them because, as they are given, they are only consistent provided the first table (A) is given in ergs per ½ second. But as the figures in Table B, being expressed in horse-power, cannot be given in ½ seconds (as the horse-power unit carries in itself its time unit and can only be used in connection with it, namely, 1 second), the figures of Table A must first be transformed into ergs per second, that is, multiplied by 2, to be identical with those of Table B. This is what has probably escaped the attention of Broca, and of Henri and des Bancels, and unfortunately, Broca took his figures from Table B, and Henri and des Bancels took theirs from Table A, so that all the figures of Broca are exactly double those of Henri and des Bancels. It is possible that these authors have not been mistaken, and that one of them (Broca) reduced the figures in Table A to ergs per second, whereas Henri and his coworker simply took them as they were. But it is most regrettable that none of them gave any indication as to the unit of time. Moreover, an important error is to be found in the figures of Henri and des Bancels due perhaps to misprint: for the wave length 0.55μ, they quote 3.0 × 10⁻⁸ ergs, (7) instead of exactly 2.77 × 10⁻⁸. If 3 may be taken as a roughly rounded figure for 2.77, however, the order of magnitude is different. In Broca’s quotation, another error or misprint is also to be found: 3.6 × 10⁻³ ergs for 0.75μ, instead of 2.56 × 10⁻².

It may be of interest to compare the tables published by Broca, and Henri and des Bancels with the exact figures of Langley:

<table>
<thead>
<tr>
<th>Wave Length (μ)</th>
<th>Langley (ergs per ½ sec)</th>
<th>Henri and des Bancels (ergs)</th>
<th>Broca (ergs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40μ</td>
<td>1,500,000</td>
<td>6.7 × 10⁻⁷ ergs/sec</td>
<td>1.33 × 10⁻⁴ ergs/sec</td>
</tr>
<tr>
<td>0.55μ</td>
<td>360,000,000</td>
<td>2.77×10⁻⁸ ergs/sec</td>
<td>5.55×10⁻⁴ ergs/sec</td>
</tr>
<tr>
<td>0.65μ</td>
<td>1,600,000</td>
<td>6.29×10⁻⁷ ergs/sec</td>
<td>1.26×10⁻⁴ ergs/sec</td>
</tr>
<tr>
<td>0.75μ</td>
<td>780</td>
<td>1.23×10⁻⁶ ergs/sec</td>
<td>2.56×10⁻³ ergs/sec</td>
</tr>
</tbody>
</table>

² Langley, (2), pp. 12, 13, 15.
Langley gives another series of figures (2) by which he intends to express "the proportionate results for seven points in the normal spectrum, whose wave lengths correspond approximately with those of the ordinary color divisions, where unity is the amount of energy (about $\frac{1}{1000}$ erg) required to make us see light in the crimson of the spectrum near A." According to this definition, this scale corresponds to the minimum visible.

<table>
<thead>
<tr>
<th>Wave lengths</th>
<th>0.40</th>
<th>0.47</th>
<th>0.53</th>
<th>0.58</th>
<th>0.60</th>
<th>0.65</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity (visual effect)</td>
<td>1.000</td>
<td>0.020</td>
<td>100.000</td>
<td>28.000</td>
<td>14.000</td>
<td>1.200</td>
<td>1</td>
</tr>
</tbody>
</table>

Expressed in negative powers of 10, in order to facilitate comparison, we have (unity being $10^{-3}$ ergs, no indication being given concerning the time):

<table>
<thead>
<tr>
<th>Wave lengths</th>
<th>0.40</th>
<th>0.53</th>
<th>0.65</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>$6.2 \times 10^{-7}$</td>
<td>$1 \times 10^{-8}$</td>
<td>$8.3 \times 10^{-9}$</td>
<td>$1 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

The first figure (for 0.40 $\mu$) agrees well with that given by Langley (2) in his other tables. The second one (0.53 $\mu$) does not agree at all, and the slight difference in wave length cannot be regarded as the cause of the discrepancy. The third one agrees within 25 per cent and the last one also, approximately. We see no explanation for this discrepancy, which cannot be due to a misprint.

Therefore, it was desirable to settle the question, since Langley's data are so misleading that good authors have made errors simply in quoting them. Quite recently, Joly (8) published a very interesting article on a quantum theory of vision, and although he does not share Henri's opinion on the subject, quotes one of his figures, $5 \times 10^{-12}$ ergs for the threshold of sensitivity for white light. Now, we have tried in vain to find such a figure in two of the papers of Henri and des Bancels, as the indications of the source are missing. As far as we know, they did not make any measurements themselves, but simply quoted those of Grijns and Noyons. They quote the figures given by Grijns and Noyons, $4.4 \times 10^{-11}$ ergs. Even if we admit that only 10 per cent of the energy is radiated under the form of light (9), we obtain $3.96 \times 10^{-12}$, and not $5 \times 10^{-12}$. It is regrettable that Professor Joly did not give the bibliographic reference.

Method.

An integration method was used. In other words, a curve representing the intensities of the dispersed beam after its passage through the prism was plotted in function of the wave lengths on coordinate paper. It is clear that the area delimited by this curve and certain
limits, arbitrarily chosen, that is to say, the integral of the curve between these limits, will express the total energy radiated. As the source yields in the same time invisible and visible rays, and as the methods used for measuring the radiation give us figures corresponding to the total radiation, \( R \) (visible + invisible), a segment extending between the limits of the visible spectrum must also be integrated. This latter integration gives the quantity of energy spread in the visible part of the spectrum; let it be \( L \). Then the ratio of these two areas will be the luminous efficiency of the source, and will be expressed by \( \frac{L}{R} = E \). The percentage of the visible to the invisible is now known. Let us call \( I \) the intensities in function of the wave lengths, \( \lambda_1 \) the lower limit of integration, \( \lambda_2 \) the upper limit of integration for the visible, then:

\[
E = \frac{L}{R} = \frac{\int_{\lambda_1}^{\lambda_2} I d\lambda}{\int_{\lambda_0}^{\infty} I d\lambda}
\]

The quantity of energy spread by the slit over the visible spectrum being thus known, a suitable screening of each monochromatic light decreases its intensity until the threshold of sensitivity is reached. Knowing exactly the amount of energy absorbed by the screens, the amount which is allowed to pass may be calculated easily: it is the minimum energy necessary to produce visual sensation.

**Technique.**

**Limits of Integration.—Limits of Total Radiation.—Lower limit:** For most light sources, the energy in the ultra-violet is so small that the lower limit, 0.4 \( \mu \), may be taken as zero without any appreciable error. Gage (10) takes it as the limit in his study of the electric arc, which is one of the richest sources in ultra-violet. The Nernst lamp, on the contrary, yields very little ultra-violet radiation, and it was assumed that this limit could safely be taken. **Upper limit:** The plotting of energy distribution curves showed that above 7\( \mu \) in the infra-red, the amount of energy radiated by the Nernst
Lamp was very small, as compared to that radiated beyond. From 7 \( \mu \) to 10 \( \mu \), it amounts to less than 1 per cent of the total. As the other errors involved by the method are of a greater order of magnitude, it was adopted as the upper limit, for the total radiation.

Limit between the Red and the Infra-Red.—(Upper limit of integration of the visible spectrum). Langley, although he does not specify it, seems to have chosen 0.75 \( \mu \) as the upper limit. Many workers have chosen 0.8 \( \mu \) (as the eye is sensitive to the radiations up to 0.8 \( \mu \)). Some have preferred 0.76 \( \mu \), others 0.7 \( \mu \). The reason for the importance of this determination is that energy increases very much between 0.7 \( \mu \) and 0.8 \( \mu \), whereas the impression on the eye is very slightly changed. In other words, the shifting of the limit from 0.8 \( \mu \) to 0.7 \( \mu \) will change considerably the amount of energy spent in the visible spectrum, whereas the effect on the eye will hardly be noticeable, since it only brings in very faint, deep red rays which, if absent, do not modify one's impression appreciably. On the other hand, if it is sought to determine the minimum of energy necessary to make the red rays between 0.7 \( \mu \) and 0.8 \( \mu \) impress the retina, one has to shift the limit as high as 0.8 \( \mu \). And in this case, all the values given for the energy of radiations below 0.7 \( \mu \) will be altered (by more than 27 per cent). Therefore, in this paper, the two figures are given, so that one may compare the results.

The study of luminosity curves shows that, by removing the part of the spectrum extending beyond 0.7 \( \mu \), the total luminosity is only decreased by 0.4 per cent. As König and Brodhun (11) have shown that the human eye was just able to detect a change in luminosity when it amounted to 1.6 per cent, we feel that this limit is advisable.

Measurement of Total Radiation.

The first step was to measure the value in absolute units of the total radiation of the Nernst lamp, with which it was intended to experiment; for it was difficult, owing to the discrepancies found in the figures given by different authors, Lux (12), Hartman (13), Ingersoll (14), etc., to rely upon data found in literature.

The source was an ordinary Nernst lamp, (110 volts, 1.3 amperes). In order to prevent any fluctuations due to cooling by air currents, the glower was enclosed in a brass chamber, with just one rectangular
slit (20 + 3 mm.), in front of the glower. The reflection from the heater and porcelain support was suppressed by fixing the glower by means of its platinum wires at the end of two leads. The inside of the chamber was blackened with soot. A voltmeter was placed across the terminals, and an ammeter in series, so as to know exactly the input in watts. Under normal conditions, it was found to be 87.5 volts \times 1.05\text{ amperes} = 91.875\text{ watts}. These 91.875\text{ watts} are not all transmitted by radiation, but part of them are taken away by conduction and convection by the air. Lux and others give the ratio \frac{\text{input}}{\text{radiation}}. But as these figures may correspond to different types of lamps, it was found safer to measure it directly. Besides, this would allow us, by a simple calculation, to check our radiation data against those published previously on the Nernst lamp.

It was first attempted to use a specially made mercury thermometer, with a known weight of mercury in a known weight of glass, blackened on the bulb of which the rays emerging from a 0.1 sq. cm. slit were concentrated by means of a fluorite lens of short focus. This process showed a lack of sensitivity and it was necessary to check it by means of an electric method. Although less difficult to handle than a bolometric device, the following apparatus required a great deal of care and time. A thermopile was made of copper and constantan wires, with ten elements, disposed linearly; the cold ends were simply bent out. On the top of the welded ends, carefully planed and ground, a thin piece of tin-foil exactly 1 mm. wide and 1 cm. long was applied and fixed with a very thin layer of shellac. Then the tin-foil was cut carefully between the welded ends, leaving a little square table of very nearly 1 sq. mm. on each thermocouple. These were blackened with soot, and the whole thermopile fixed in a thermostat. The rays were allowed to fall on the pile through an adjustable slit, and the distance between the source and the couples made equal to 1 meter. The method consisted in compensating the heat generated by the incoming radiation, by the current sent in a strip of constantan placed near the cold ends of the thermocouples, in a tiny calorimeter, 2 cc. in capacity, filled with oil, and well isolated. The following formula was used:

\[ E = Kp \frac{\text{cal. gr.}}{0.1\text{ sq. cm. sec.}} \]
$K$ being a constant (function of the resistance of the constantan strip) of the instrument, calculated and experimentally checked, equal to 0.21, we measured a current of 0.0028 amperes; this gives:

$$0.21 \times (0.0028)^2 \frac{\text{cal. gr.}}{\text{sec.} \ 0.1 \ \text{sq. cm.}} = 0.00000165$$

as 0.2388 cal. gr. = 1 watt sec., it corresponds to 0.000694 watts by sq. cm.

*Correction for Equatorial Radiation.*—This corresponds to the homogeneous radiation of a punctual source of energy of 87 watts; that is, it would require 87 watts from a punctual source to radiate spherically in all directions an amount of energy of that magnitude. We have measured this amount equatorially, that is, normally to a line normal to the glower itself, and, of course, in the best conditions of radiation. But as the beam of light assumes a greater deviation from the equatorial plane, in the case of an incandescent rod, in other words, as the square centimeter exposed to the rays stands higher in latitude on the sphere, the amount of energy radiated is decreased, since the rays are no longer emitted perpendicularly by the rod. Around the two poles, there is even a region where there is no radiation at all. The result is that, whereas the source acts as radiating 87 watts equatorially, it radiates much less as soon as we reach higher latitudes, and becomes zero at the poles, and the mean value of the radiation is much less than 87 watts. It is known that by multiplying the energy radiated equatorially by $\frac{\pi}{4}$, the real value of the radiating energy from the source is known. In this case, $87 \times \frac{\pi}{4} = 68.3$ watts.

Hence, out of the 92 watts sent into the filament, only 68.3 are radiated, and 23.7 are lost by convection and conduction. The ratio $\frac{92}{68.3} = 1.35$ is in excellent agreement with the figure given by Lux: 1.34.

This figure may be checked in another way: the input in the filament being 91.8 watts, roughly 92 watts, it will radiate equatorially $92 \times \frac{4}{\pi} = 118$ watts,
The quantity radiated actually as measured equatorially, being approximately. The quantity radiated actually as measured equatorially, being 87 watts, the ratio $\frac{118}{87} = 1.35$ gives the amount of energy lost.

*Calculation of the Luminous Efficiency, and Corrections.*—The next step was to determine the ratios

$$\frac{\int_{0.7}^{0.4} L d\lambda}{\int_{0.4}^{0.7} R d\lambda} \text{ and } \frac{\int_{0.4}^{0.8} L d\lambda}{\int_{0.4}^{0.4} R d\lambda} = \frac{L}{R}$$

This ratio has been calculated by many authors, Lux, Nichols and Coblentz (15), Ingersoll, Ångström (16), Stewart and Hoxie (17), etc. Their methods were different and their results do not always check perfectly, (some varying by more than 50 per cent, for example, those of Lux and Ångström.) Some of the workers used methods based upon the absorption of one part of the spectrum by water cells in which different substances were dissolved, (copper sulphate, iodine). It has been shown by Nichols and Coblentz that none of these methods based on absorption were reliable. Ingersoll studied the Nernst lamp and published figures of observed luminous efficiency, which vary greatly according to different lamps, and besides, correspond to burners whose consumption was not that of our lamp, (89 watts). Therefore, the energy distribution curve of our burner was plotted by means of a Hilger Infra-Red Spectrometer.\(^5\)

*Correction for the Non-Normal Spectrum.*—The spectrum was corrected for the lack of homogeneity of the dispersed beam. Indeed, the refracted rays are contracted in certain parts of the spectrum, and expanded in others, so that, for instance, the same slit opening (e.g., 0.25 mm.) covers 0.015 $\mu$ on the spectrum at a mean wave length 0.68 $\mu$ (from 0.6725 $\mu$ to 0.6875 $\mu$), and as much as 0.266 $\mu$, more than 17 times as much, at the mean wave length 2.66 $\mu$ (from 2.53 $\mu$ to 2.79 $\mu$). This correction was introduced by the consideration of the geometry of the screw motion, (pitch of screw in relation to

\(^5\) Instrument No. 281, Rock salt prism, Angle 59°. 57'. 30'.
rotation of the prism table), and the use of the dispersion formula, given by Paschen (18):

\[ n^2 = a^2 + \frac{M_1}{\lambda^2 - \lambda_1^2} + \frac{M_2}{\lambda^2 - \lambda_2^2} - K\lambda^2 - \lambda^4 \]

Then, the range of the spectrum embraced by a given slit was checked by moving a spectrum line across the slit, and reading the result on the drum. The right edge of the D lines (sodium), for instance, was brought in contact with the right edge of the slit (0.25 mm. opening), and the reading made. Then it was moved toward the left until the whole D line just disappeared, and another reading made. The result was 0.008 μ. This was done for the lines of copper (0.4955 μ, 0.5292 μ), mercury (0.5461 μ), sodium (0.5893 μ), and cadmium (0.6439 μ). For the infra-red, the data are published by Hilger (19).

It was decided to take the area covered at 2.66 μ by a slit 0.025 mm. wide as unit, (0.026 μ) and to fix the slit in such a way, for every wave length, as to cover the same range. A high sensitivity Leeds and Northrup galvanometer was used in connection with the thermopile, (galvanometer resistance = 12 ohms).

Corrections of the Absorptions Due to the Spectrometer.—But, before integrating the plotted curve, another very important correction had to be introduced regarding the absorption by the golden mirrors, because the energy distribution curve does not correspond to the total amount received by the collimator slit, and because the absorption is much greater for short than for long wave lengths. Fig. 1 shows how the absorption varies for different wave lengths.

If \( R \) is the coefficient of reflection, and \( S \) the number of mirrors, the amount of energy reflected is expressed by

\[ I = I_0 R^S \]

It is easily seen that for 0.5 μ, for example, only \( \frac{1}{9.6} \) of the incident light is transmitted, and much less still for 0.4 μ.

The absorption by the prism amounts to very little. Theoretically, from the formula, \( J = J_0 K^c \), where \( c \) is the length of the path of light in the rock salt and \( K \) a constant equal to 1 between 0 and 9 μ, it is equal to zero. We found that, practically, for the visible,
Fig. 1. Reflection of monochromatic light by gold.

Fig. 2. Energy distribution curves for Nernst lamp, corrected and uncorrected for mirror absorption. Outside curve corrected; inside curve uncorrected.
it really amounts to 1.5 per cent approximately, by concentrating
the beam of light before and after passing through the prism.

Finally, the curves were integrated graphically. Fig. 2 shows
the different aspects of the curves before and after corrections due
to absorption. (These curves are not to scale, in order to emphasize
the difference.) The results were:

\[ R = 110.3, \quad L_{0.4} = 4.8, \quad L_{0.7} = 3.6 \]

The ratios are:

- Upper limit = 0.8, \[ \frac{L}{R} = 0.0435 \]
- Upper limit = 0.7, \[ \frac{L}{R} = 0.0316 \]

These results are in good agreement with those of Ingersoll, who,
with the upper limit 0.76, found figures between 0.036 and 0.046,
and in contradiction with those of Drude (20), who gives 12 per cent
as luminous efficiency of Nernst lamps, (instead of 4.35 per cent).
Lux gives 5.96 per cent, but takes no account of the fact that the
screening method he used allowed most of the radiation up to 1.2 \( \mu \)
to pass. Coblentz and Nichols found 0.033 for the efficiency of acety-
ylene flame.

Hence, as we have established that our source radiated 87 watts
equatorially, the quantity radiated as luminous waves is equal to
4.35 per cent of 87 (limit 0.8 \( \mu \)) = 3.8 watts, and 3.16 per cent of
87 (limit 0.7 \( \mu \)) = 2.75 watts. Let us take the quantity corresponding
to the limit 0.7 \( \mu \) for example. These 2.75 watts are radiated at the
distance of 103 cm. from the source, (distance of the thermopile)
by square centimeter. The energy then becomes 0.0000206 watts.
The area of the slit being 0.1 sq. cm., it only receives 0.00000206
watt seconds, or \( 20.6 \times 10^{-7} \) watts = 20.6 ergs. This quantity
of energy is spread over the range delimited by the upper and lower
limits of integration (0.4 \( \mu \) to 0.7 \( \mu \)) and the energy distribution
curve corrected for the absorption by mirrors, and by the prism,
that is, over a surface of 83 sq. cm., (obtained by graphical integration
of the curve, Fig. 3). This corresponds, in the scale chosen, to
0.248 ergs by unit of surface. Similarly, between the limits 0.4 \( \mu \)
and 0.8 μ, we find 0.343 ergs by unit of surface. In order to determine the efficient amount of energy which will affect the eye, it will easily be seen that this quantity will only have to be multiplied by the segment delimited on the same chart, by the two ordinates corresponding to the range covered by the slit on the spectrum and the energy distribution curve uncorrected for mirror absorption.

![Energy distribution curve](image)

**Fig. 3.** Energy distribution curve, corrected and uncorrected, in a larger scale in the visible part of the spectrum. A B represents the area covered by a slit 0.25 mm. wide.

**Test of the Eyes.**

In order to reduce the intensity of light by a known quantity, a set of absorbing screens was prepared carefully. It was sought to look directly into the beam of light instead of using reflected light, in order to avoid the errors arising from the reflection of very faint radiations. By getting screens which could decrease by the same known amount, for example, 90 per cent, the intensity of the incident light, the simple formula

\[ I = I_0 K^{-n} \]
in which \( I = \) emergent light, \( I_0 = \) incident light, \( K = \frac{I_0}{I} \) and \( n = \) the number of screens interposed, leads to this:

\[
\log I_n = \log I - n \log K
\]

and as \( K = 10 \)

\[
\log I_n = \log I_0 - n
\]

whence

\[
I_n = I_0 10^{-n}
\]

The number of screens interposed will itself give the order of magnitude of the out-coming energy: 2 screens will mean that the energy is decreased by 100; 4 screens, by 10,000, etc.

It was found that especially prepared white paper fulfilled the requirements better than any other screen. Sheets of the same paper were chosen, (mean thickness 0.09 mm.), and placed exactly in front of the thermopile, then the throw of the galvanometer was observed; the paper was removed and another reading made at three different wave lengths. A great number were tested, and as we were unable to get ten sheets exactly similar, the thicker were placed on a plane surface and evenly rubbed down with very thin sand, then glossed again with a piece of round glass. They were frequently tested during this process, and finally the following results were considered as satisfactory. (The figures express the ratio \( \frac{I}{I_0} \)).

**TABLE II.**

<table>
<thead>
<tr>
<th>Sheets</th>
<th>0.55μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( \frac{I}{I_0} )</td>
</tr>
<tr>
<td>1</td>
<td>0.100</td>
</tr>
<tr>
<td>2</td>
<td>0.103</td>
</tr>
<tr>
<td>3</td>
<td>0.103</td>
</tr>
<tr>
<td>4</td>
<td>0.095</td>
</tr>
<tr>
<td>5</td>
<td>0.0955</td>
</tr>
<tr>
<td>6</td>
<td>0.105</td>
</tr>
<tr>
<td>7</td>
<td>0.100</td>
</tr>
<tr>
<td>8</td>
<td>0.0965</td>
</tr>
<tr>
<td>Mean value</td>
<td>0.1005</td>
</tr>
</tbody>
</table>
This particular paper was less transparent for the extreme red than for the green, a fact which had to be taken into account.

Measurements.—Eighteen persons were examined; two series of experiments were performed: one after 8 minutes in the dark, and one after 25 minutes. Only five persons were examined for all wave lengths. The others were merely tested for the radiation 0.55. The measurements were carried on in the following way: When nine sheets of paper were placed exactly against the slit, generally no light could be seen. Then, one after the other, the sheets were removed, according to the intensity of light, and usually, owing to the relatively large area (0.1 sq. cm.), there was no difficulty whatever in determining the order of magnitude of the minimum visible. Namely, one sheet added gave a black impression, and this sheet removed left a visible, although very faintly colored, image of the slit. We sought, as Langley did, to determine the minimum visible, defining this to be, not the smallest light whose existence it is possible to suspect, or even to be reasonably certain of, but a light which is observed to vanish and reappear when silently occulted and restored by an assistant without the observer's knowledge (Fig. 4).

On top of the last sheet of paper, another slit was placed, across the first one. Its jaws were cut in such a way that a square opening was left between them, (see Fig. 5); thus a square or rectangular figure was delimited by four moving lines. At first, the slit was adjusted so as to cut a little window of 1 sq. mm. on the luminous spot. If the window could not be seen, the jaws of the slit were moved micrometrically until the spot became visible. The maximum opening corresponded to a vertical motion of 5 mm.

It was found that most women generally require more time than men to reach the same degree of sensitivity. Most of the men tested became adapted in 5 minutes (viz., could see a light corresponding to an energy of the order of magnitude $10^{-9}$) while it required 15 to 20 minutes for women to see the same thing. Moreover, some of them could only see the spot spasmodically appearing and disappearing, while men had a continuous impression. Only an increase in the intensity of about 50 to 100 per cent was able to give them the same visual impression. As will be seen, the differences vary between 0 and 25 per cent among men for a given wave length;
Fig. 4. Apparatus set for the test of the eyes. The observer was looking through slit S. L, Near lamp; M, motor; R, S, revolving shutter; A, ammeter; V, voltmeter; S, S, slides; P, rock salt prism; D, calibrated drum; m, m, mirrors; K, lamp and scale.
among women, between 0 and 100 per cent. One man showed a marked difference.

The figures express the energy necessary for continuous impression but by making the same assumption as Langley concerning the

minimum amount of time necessary to perceive distinctly a very faint light, (about ½ second), these figures may be expressed in ergs per ½ second, by dividing them by 2.

The size of the retinal image was approximately 0.01 sq. mm. ±0.002.

### TABLE III.

#### Men.

<table>
<thead>
<tr>
<th>Observer</th>
<th>Age</th>
<th>0.4 to 0.8 μ</th>
<th>0.4 to 0.7 μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>29</td>
<td>7.1×10⁻⁹</td>
<td>4.55×10⁻⁹</td>
</tr>
<tr>
<td>B</td>
<td>36</td>
<td>8.2×10⁻⁹</td>
<td>5.9×10⁻⁹</td>
</tr>
<tr>
<td>C</td>
<td>45</td>
<td>7.1×10⁻⁹</td>
<td>4.55×10⁻⁹</td>
</tr>
<tr>
<td>D</td>
<td>36</td>
<td>8.3×10⁻⁹</td>
<td>5.9×10⁻⁹</td>
</tr>
<tr>
<td>E</td>
<td>28</td>
<td>8.3×10⁻⁹</td>
<td>5.95×10⁻⁹</td>
</tr>
<tr>
<td>F</td>
<td>23</td>
<td>8.1×10⁻⁹</td>
<td>5.95×10⁻⁹</td>
</tr>
<tr>
<td>G</td>
<td>19</td>
<td>8.6×10⁻⁹</td>
<td>6.1×10⁻⁹</td>
</tr>
<tr>
<td>H</td>
<td>30</td>
<td>7.4×10⁻⁹</td>
<td>5.25×10⁻⁹</td>
</tr>
<tr>
<td>I*</td>
<td>27</td>
<td>1.0×10⁻⁹</td>
<td>8.6×10⁻⁹</td>
</tr>
</tbody>
</table>

Mean values. ........................................ 8.1×10⁻⁹  5.85×10⁻⁹

* Observer "I" was included in the mean, although he seemed to be quite out of the normal.
First Series of Experiments.—The observers were protected by a curtain from all light, and waited until their eyes had become quite sensitive before making the experiments. 8 minutes in absolute darkness seemed to be sufficient for men. These first figures will show the difference between the rapidity of adaptation of men and women. Wave length 0.55 \( \mu \) (Tables III and IV).

### TABLE IV.

<table>
<thead>
<tr>
<th>Observer</th>
<th>Age.</th>
<th>0.4 to 0.8 ( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>30</td>
<td>Normal sight.</td>
</tr>
<tr>
<td>K</td>
<td>30</td>
<td>Short sighted.</td>
</tr>
<tr>
<td>L</td>
<td>25</td>
<td>&quot; &quot;</td>
</tr>
<tr>
<td>M</td>
<td>22</td>
<td>Normal sight.</td>
</tr>
<tr>
<td>N</td>
<td>24</td>
<td>&quot; &quot;</td>
</tr>
<tr>
<td>O</td>
<td>23</td>
<td>&quot; &quot;</td>
</tr>
<tr>
<td>P</td>
<td>19</td>
<td>&quot; &quot;</td>
</tr>
<tr>
<td>Q</td>
<td>42</td>
<td>&quot; &quot;</td>
</tr>
<tr>
<td>R</td>
<td>40</td>
<td>&quot; &quot;</td>
</tr>
</tbody>
</table>

Obviously, for women the differences are so great that a mean value would have no significance at all.

It was found that it took over 20 minutes for observers J, M, and O to reach the same sensitivity as men, viz., less than \( 7 \times 10^{-9} \). As Langley does not give any precision concerning the time of adaptation, we may compare his figures to the mean value found for men:

Langley (0.55 \( \mu \)), \( 5.55 \times 10^{-9} \) ergs/sec. We found slightly larger figures:

\[
\int_{0.4}^{0.7} : 5.85 \times 10^{-9} \text{ ergs/sec.}
\]
\[
\int_{0.4}^{0.8} : 8.1 \times 10^{-9} \text{ ergs/sec.}
\]

But after more than 20 minutes in the dark, the eye becomes more sensitive still, and we obtained the following figures (Table V). These figures are smaller than those given by Langley, but as he did not state the length of time which the eyes of his experimenters
were kept in the dark, and as we have seen that the sensitivity is increased over 100 per cent by a stay of 25 minutes instead of 8 or 10, they cannot well be compared. Generally, at least, they are of the same order of magnitude for the wave length 0.55 μ. A stay of 1 hour in absolute darkness did not seem to increase the sensitivity beyond these figures.

It must be pointed out that the figures corresponding to the wave length 0.4 μ are doubtful, as the spectrometer which was used was not fit for the measurements in that part of the spectrum, owing to the gilded mirrors. They are only given as approximations. It must also be borne in mind that these quantities of energy do not correspond exactly to one pure radiation of wave length, 0.55 μ for instance, but to the beam comprised between 0.537 μ and 0.563 μ, the slit covering a range of 0.026 μ.

**White Light.**—The same technique was applied to the minimum visible for white light (Nernst lamp), and gave 3.8 × 10⁻¹¹ ergs, for continuous impression by total radiation. This figure agrees well with that of Grijns and Noyons for the Hefner lamp, 4.4 × 10⁻¹¹. It is better to compare figures related to total radiation, because these figures do not involve the more or less arbitrary choice of limits of integration and the knowledge of the ratio \( \frac{L}{R} \) for the considered source.

### TABLE V.

*25 Minutes in the Dark. \( \int_{0.4}^{0.8} \)

<table>
<thead>
<tr>
<th>Observers</th>
<th>0.4</th>
<th>0.5</th>
<th>0.55</th>
<th>0.65</th>
<th>0.68</th>
<th>0.72</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(Women)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>2.5×10⁻⁶</td>
<td>1.6×10⁻⁶</td>
<td>3×10⁻⁷</td>
<td>2×10⁻⁷</td>
<td>2×10⁻⁷</td>
<td>3×10⁻⁷</td>
</tr>
<tr>
<td>O</td>
<td>5×10⁻⁶</td>
<td>2.3×10⁻⁶</td>
<td>4.5×10⁻⁷</td>
<td>3.5×10⁻⁷</td>
<td>5×10⁻⁷</td>
<td>4×10⁻⁷</td>
</tr>
<tr>
<td><strong>(Men)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>5×10⁻⁷</td>
<td>1.4×10⁻⁷</td>
<td>2×10⁻⁷</td>
<td>1.5×10⁻⁷</td>
<td>1.9×10⁻⁷</td>
<td>2×10⁻⁷</td>
</tr>
<tr>
<td>D</td>
<td>5×10⁻⁷</td>
<td>1.3×10⁻⁷</td>
<td>2×10⁻⁷</td>
<td>1.5×10⁻⁷</td>
<td>2×10⁻⁷</td>
<td>2×10⁻⁷</td>
</tr>
<tr>
<td>F</td>
<td>8×10⁻⁷</td>
<td>1.5×10⁻⁷</td>
<td>3×10⁻⁷</td>
<td>1.7×10⁻⁷</td>
<td>2×10⁻⁷</td>
<td>2.5×10⁻⁷</td>
</tr>
<tr>
<td><strong>Mean values...</strong></td>
<td>3.85×10⁻⁷</td>
<td>1.6×10⁻⁶</td>
<td>3×10⁻⁷</td>
<td>2.2×10⁻⁷</td>
<td>2.7×10⁻⁷</td>
<td>2.5×10⁻⁷</td>
</tr>
</tbody>
</table>
Fig. 6. Energy distribution curve of the Nernst lamp used (uncorrected), 91.8 watts: slits: 0.025 mm.

Criticisms of the Results.—As has been pointed out before by Langley, the errors involved in the determination of the threshold of sensitivity (*minimum visible*) may be perhaps 100 per cent, or even more. For this reason, the absorption by the various eye
media, for the total depth of the eye, which amounts only to about 1 per cent for 0.7 μ and less than 0.1 per cent below 0.65 μ, are entirely negligible. The eye, for such small amounts of energy as those corresponding to the minimum visible, does not perceive a continuous increase in the brightness of the spot, when its intensity is increased progressively, but seems to react by steps. High authorities, such as Joly, and Henri, disagree entirely as to the explanation of vision on the basis of the quantum theory.

Accuracy of the Method.—In the method used, the following causes of error could be corrected:

**Errors Due to the Spectrometer.**

1. Selective reflection by the three gilded mirrors.
2. Selective absorption by the rock salt prism.

**Errors Due to the Use of a Nernst Filament.**

1. Uneven distribution of spherical energy.
2. Disturbing effect of volt- and ammeter in the circuit of the glower.

The following errors are also involved, and were not corrected:

**Errors Due to the Integration Method.**

2. Material errors due to the mechanical integration of surfaces.

We can probably admit that they do not amount to more than 10 per cent.

**Errors Due to the Assumed Quantity of Energy Radiated.**

1. Errors due to the fact that the image of the glower was not formed on the collimator slit.
2. Errors due to the emission of radiations from other parts of the instrument.
3. Errors due to the measurement of the total radiation by means of a compensating current.
As our figures are in good accord with those of the best authors, within less than 10 per cent, we may assume that this is the upper limit of error. This gives a total of 20 per cent possible error, which is beyond the possibility of detection by the eye in the minimum visibile, as stated before. As some of the individual data differ by more than 100 per cent, the data can only be considered as reliable in the conditions of the experiments, within about 120 to 150 per cent. This is about the order of magnitude of the differences between the experimental data given by Langley.

Quantum Theory.—We can roughly express the minimum visibile in function of quanta of energy. For the mean radiation 0.55 μ, the period of the atom is $5.76 \times 10^{14}$ per second. The minimum of energy perceived is approximately equal to $1.9 \times 10^{-12}$ ergs per second, (taking $3.8 \times 10^{-11}$ as the value of the minimum for total radiation, and roughly 5 per cent as belonging to the visible spectrum). Hence,

$$\frac{1.9 \times 10^{-12}}{5.75 \times 10^{14}} = 3.3 \times 10^{-37}$$

As Planck’s universal constant $\hbar = 6.5 \times 10^{-37}$, the figure found is satisfactory as far as the order of magnitude is concerned, but it would mean that only one-half quantum per second would be sufficient to cause the luminous sensation; as we have dealt with an area of $\frac{1}{100}$ of a square millimeter on the retina, it would indicate that the destruction of one molecule every 2 seconds on such an area would be sufficient to produce an impression of light.

CONCLUSION.

A method was devised for measuring the minimum visibile in different parts of the spectrum, as done by Langley in 1888. The results are generally in good agreement with those given by this author, although not as close on both sides of the wave length 0.55 μ; this may be due partly to the use of a rock salt prism, to the fact that the minimum was determined by looking at a beam of diffused transmitted, instead of diffused reflected light, and also to the fact that Langley experimented with the sun, through the earth’s atmosphere, and had to take into account the thickness of the atmos-
sphere interposed and the brightness of the sky. Although his experiments were made with great care, the differences from one day to another are important. However, when he expresses the energy in absolute units, he always refers to the same mean amount of energy radiated by the sun on 1 sq. cm. This amount is certainly not constant, if one judges from the differences observed in two measurements of sensitivity of the eye of the same individual at different dates. On the contrary, for a given wave length, our measurements always agreed closely, as our source of radiation was very nearly constant, owing to the absence of a varying amount of water vapor interposed. This may in some way account for the discrepancies observed.

I wish to express my thanks to Dr. Harry Clark of The Rockefeller Institute for the valuable help he was kind enough to give me in solving certain difficulties which were encountered during this work.

BIBLIOGRAPHY.

2. Langley, S. P., Phil. Mag., 1889, xxvii, 1.
   Issued with instrument.