AN APPLICATION OF AUTOCORRELATION METHODS TO THE INTERPRETATION OF INTESTINAL MOTILITY RECORDS*

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INTRODUCTION

Many physiologic functions fluctuate with time and the wave forms vary in complexity. Simple waves, such as the peripheral pulse, present no serious problem in analysis. When, however, the wave form is complex in appearance, existing analytic methods do not yield precise information. The general similarity between complex physiologic wave patterns and communication signals, such as sound waves, suggested that a method of statistical analysis recently developed in the field of communications might be applied to these complex waves. In the application of this method a complex wave is reduced mathematically to a simpler form from which the periodic and random components can be isolated and statistically evaluated. The periodic component is sinusoidal; the random component is irregular in form, and has no definite frequency. Any random function is known to consist of the sum of many sine waves with no particular relationship, while the periodic component usually consists of a few (sometimes only one) sine waves of harmonically related frequencies. Although the random component is sometimes considered to be useless "noise," at other times it may contain valuable information.

An intestinal motility record is an example of a complex physiologic wave pattern which has been difficult to interpret. In this paper, the communications method has been applied to the analysis of records of intestinal motility in rabbits.

Four motility records were analyzed in this study and for each record we have derived quantitative expressions for total intestinal activity and for frequency, amplitude, and variations of amplitude of the periodic components. In addition, a measure of the total random activity was obtained as well as the range of frequencies and relative amplitudes of the sine waves making up

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this random component. The quantitative expressions obtained permit comparison of intestinal motor activity recorded by different techniques and under widely varying physiological conditions. Furthermore, the individual components of motility, once isolated by this technique, can be correlated with other intestinal processes.

Method of Analysis

To investiğate the motility records, two mathematical operations called autocorrelation and Fourier transformation were borrowed from the field of communications where they are used in the statistical treatment of random and periodic functions (1, 2).

The mathematical procedure followed in the analysis of the records consists of:

1. The generation and study of the autocorrelation function.
2. The isolation and study of components of the autocorrelation function.
3. Fourier transformation of the components of the autocorrelation function.

By the use of these steps, the detection and isolation of the following components are possible:

1. The random components, their contribution to the magnitude of the total record, and their spectral composition.
2. The constant-amplitude periodic components, their magnitudes and frequencies, and their contribution to the magnitude of the total record.
3. The inconstant-amplitude periodic components, their magnitudes and frequencies, their contributions to the magnitude of the total record, and their spectral composition.

A brief description and interpretation of autocorrelation functions follow:

Autocorrelation.—Previous experience has shown us that wave forms of the type usually found in records of intestinal motility may be regarded as being composed of the sum of three basic wave forms. One component wave form is sinusoidal in nature and is completely predictable, a second component is also sinusoidal in nature but of inconstant amplitude, and a third component is random in nature. The process of autocorrelation transforms a given data record into a new form which allows easy recognition and separation of the basic components.

In autocorrelation, a given wave form is multiplied by the same wave form shifted by a given time interval, τ, as shown in Fig. 1 a. The product of these two curves is another curve, shown in Fig. 1 b. The average value of this product curve constitutes one point of the autocorrelation function corresponding to a particular displacement or τ-shift. This process is repeated for other values of τ-shift. The autocorrelation function is formed by plotting a series of these points determined for different values of τ. The data in the original record, which is usually lengthy, are therefore compressed into a small graph showing
FIG. 1. The graphical determination of an autocorrelation point. The amplitude of the given wave form \( a \) is measured at equally spaced points, \( a_1, a_2, a_3, \) etc. These values are multiplied by the values measured on the displaced curve \( b \) at corresponding points in time \( b_1, b_2, b_3, \) etc. This process yields a sequence of products \( a_1b_1, a_2b_2, a_3b_3, \) etc., which when plotted form a product curve (Fig. 1 b). The average value of this product curve \( R(\tau) \) determines one point of an autocorrelation function.
magnitudes of autocorrelation plotted for various values of \( \tau \)-shift. Mathematically, an autocorrelation function is designated by the symbol \( R(\tau) \), and an autocorrelation point found for shift \( \tau_1 \) is designated \( R(\tau_1) \).

A number of other points are computed for other values of \( \tau \)-shift as in Fig. 1. The size and number of \( \tau \)-shifts required to define the autocorrelation function adequately depend on the nature of the wave form being studied.

The same procedure is followed in the analysis of the important component wave forms of a complex wave. These are the random component, the constant-amplitude periodic component, and the inconstant-amplitude periodic component.

It can be shown from the statistical theory of communications that the autocorrelation function of a random wave form designated \( R_N(\tau) \), (Fig. 2 a), will always have the form indicated in Fig. 2 b. The value of the function at zero shift \( (\tau = 0) \) is the mean square of the amplitude, \( X_Z^2 \), of the random wave form.

The autocorrelation function of a random wave is always composed of the
Fig. 3. Sine wave (a) and autocorrelation of a sinusoidal wave (b).

Fig. 4. Autocorrelation of a sinusoidal wave with random amplitude modulation.
same sinusoidal frequency components that constituted the original random wave form. The rapidity of decay of the autocorrelation function is a measure of the number of sinusoidal frequencies composing the random wave form; i.e., the more numerous and the wider the range of the component frequencies that make up a random wave, the faster the decay. Fig. 2 c shows two curves where curve y indicates a greater number of component frequencies than curve x.

It can also be shown that the autocorrelation function, \( R_p(\tau) \), of a constant amplitude sinusoidal wave (Fig. 3 a) is always a cosine wave as shown in Fig. 3 b. The value of the autocorrelation curve at \( \tau = 0 \) is the mean square amplitude, \( \bar{X}_r^2 \). The peak to peak amplitude, \( A \), of the original wave is expressed as \( A = \sqrt{8X_r^2} \). The period of the autocorrelation function is the same as the period of the sinusoidal wave.

Another wave form whose autocorrelation is easily identified is a sinusoidal wave which is randomly modulated in amplitude. The autocorrelation function of such a wave is always a decaying sinusoid as in Fig. 4. The value of the autocorrelation function at zero shift, \( \tau = 0 \), is the mean square value, \( \bar{X}_r^2 \), of the randomly modulated original wave.

When a wave form such as a record of intestinal motor activity, Fig. 5 a, containing a random component, a constant-amplitude periodic component, and an inconstant-amplitude periodic component is autocorrelated, the resulting function appears as in Fig. 5 b. This curve contains all the frequency components of the original data and is the sum of the autocorrelation functions of the three components which constitute the wave form.

The initial value of this autocorrelation function at \( \tau = 0 \) is the mean square value, \( \bar{X}_r^2 \), which represents the total activity present in the original complex wave. The autocorrelation function of each of the three basic components may be isolated and evaluated separately. This isolation can be performed by subtracting, by graphical means, the constant-amplitude periodic component, \( R_p(\tau) \), from the total autocorrelation function, \( R(\tau) \). The decaying sinusoid (inconstant amplitude periodic) of the same frequency can then be subtracted leaving the autocorrelation function of the random component.

The process of autocorrelation is defined mathematically as:

\[
R(\tau) = \lim_{\tau \to \infty} \frac{1}{2T} \int_{-T}^{T} f_i(t)f_i(t + \tau) \, dt
\]

in which \( f_i(\tau) \) is the instantaneous value of the wave form being studied (intestinal pressure) as it varies with time, and \( f_i(t + \tau) \) is the instantaneous value of the wave form being studied after it has been shifted in time by an amount, \( \tau \).

Pressure Intensity Spectra and Fourier Transformation.—Further analysis of the original recording may be accomplished by Fourier transformation of
Fig. 5. Complex wave (a) and its autocorrelation function (b).
the autocorrelation function. By means of this mathematical procedure, a pressure intensity spectrum is obtained. A pressure intensity spectrum displays in graphic form the magnitudes and frequency range of the sinusoidal components that make up the autocorrelation function and the original record.

Each sinusoidal component of the original record is related to the total character of the original record as an overtone is related to a note from a musical instrument. The harmonics and overtones of violin notes, for example, are actually sinusoidal components of those notes and serve to give timbre or character to the violin which distinguishes it from other instruments. If the wave form corresponding to one violin note were analyzed by Fourier transformation, these distinguishing sinusoidal components as well as the basic frequency of the note would be shown graphically by the intensity spectrum. Like the spectrum of a violin tone, the intensity spectrum of a motility record serves to display the character of such a record and allows comparisons between different motility records. In the work described here, the intensity spectra of the motility records were obtained by Fourier transformation of the component autocorrelations.

The intensity spectrum of an autocorrelation function is defined mathematically as:

\[ G(\omega) = \frac{1}{\pi} \int_0^{\infty} R(\tau) \cos \omega \tau \, d\tau \]

**Description of Data.**—Four records of intestinal motility were analyzed, two records from each of two rabbits. Reproductions of two of the data records are shown in Fig. 6 a and 6 b.

The detection of rabbit intestinal activity was accomplished by recording intraluminal pressures in the jejunum. A saline-filled polyethylene catheter (0.78 mm. internal diameter) transmitted the pressures to a Sanborn electromanometer (transducer). The activity was recorded on a Sanborn "twin-viso" kymograph. The kymograph was adjusted so that a pressure of 40 mm. of Hg would cause a writing stylus deflection of 38 mm. The recording tape was run at the rate of 15 cm./minute. Two 3 minute segments of the records derived from each of the rabbits were analyzed.

**Equipment for Computation.**—Autocorrelation and Fourier transformation by hand calculation are arduous and time-consuming even when performed on simple functions. In order to avoid hand calculations in this investigation, use was made of a manually controlled mechanical correlator to analyze two records, and an electronic digital computer was used to analyze two other records.

The mechanical correlator used is a computer designed and built by the Servomechanisms Laboratory of the Massachusetts Institute of Technology (3). It performs only the correlation operation and not the Fourier transformation. A transport mechanism draws the kymograph record beneath two manually operated indices which trace the curve as it passes. Each index requires an operator, and time delay is achieved by separating the indices along the record so that one index traces the data later than the other.
Fig. 6. Intraintestinal pressure records (a and b) in rabbits and the corresponding autocorrelation functions (a' and b').
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For each point on the autocorrelation curve, the entire record must be traced, and a different index separation is required for each point. The mechanical correlator handles direct data records of the form in which the original intestinal data were taken.

The digital electronic computer used is the whirlwind I (WWI), designed and operated by the Digital Computer Laboratory of the Massachusetts Institute of Technology. This computer requires that data presented to it be in digital form.

To make the directly recorded motility data compatible with the computer requirements, the amplitudes of the records were read at equally spaced intervals. This was accomplished by establishing an arbitrary horizontal reference line and determining numerical amplitude values by measuring the vertical distance to the curve from equally spaced positions along the reference line. These positions were \( \frac{1}{2} \) inch apart, and the amplitude values of the motility curves were read to the nearest hundredth of an inch. The numerical data from the records were coded and impressed on punched paper tape by WWI personnel. The tape was then submitted to the digital computer for autocorrelation.

RESULTS AND DISCUSSION

The results of the quantitative analysis of the four motility records are shown in Table I. The procedure which was followed to obtain these values is best demonstrated by a complete explication of one record. The period of normal activity (Fig. 7 a) of rabbit 1 was chosen for this purpose.

The autocorrelation function (Fig. 7 b) of this record was obtained by the use of the mechanical correlator. From this function we may read directly at \( \tau = 0 \), the mean square value, \( \langle X^2 \rangle \). This value, 9.63 mm.\(^2\) Hg, expresses the

<table>
<thead>
<tr>
<th>Subject</th>
<th>Periodic Components</th>
<th>Random components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant amplitude</td>
<td>Inconstant amplitude</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>Peak-to-peak amplitude</td>
</tr>
<tr>
<td>No. 1</td>
<td>cycles per min.</td>
<td>mm.(^2) Hg sec.</td>
</tr>
<tr>
<td>Normal</td>
<td>25.7</td>
<td>3.76</td>
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<tr>
<td>Irradiated</td>
<td>24.3</td>
<td>1.68</td>
</tr>
<tr>
<td>No. 2</td>
<td>cycles per min.</td>
<td>mm.(^2) Hg sec.</td>
</tr>
<tr>
<td>Normal</td>
<td>24.8</td>
<td>0.87</td>
</tr>
<tr>
<td>Irradiated</td>
<td>23.6</td>
<td>0.84</td>
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total activity of the original record and is the sum of the activity contributions of the component functions.

The autocorrelation function is next separated by graphic subtraction into its components: a random component, a constant-amplitude periodic component, and an inconstant-amplitude periodic component of the same frequency. These individual components may then be analyzed. The random component (Fig. 6 c) represents all non-periodic intestinal motor activity as well as intraluminal pressure changes induced by other visceral movement. This random activity makes up part of the total activity value and its contribution, $X_{\text{R}}$, is found to be 6.98 mmHg. The frequencies constituting the random component are derived by Fourier transformation and are shown in the pressure

![Graph showing pressure recording and its autocorrelation]
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The constant-amplitude periodic component (Fig. 8 a) probably represents the "segmental" contractions of the intestine. The peak-to-peak amplitude of these "segmental" waves is 3.76 mm Hg and they occur at a rate of 25.7 per minute. The contribution of this component to the total activity is 1.76 mm$^2$ Hg.

The inconstant-amplitude periodic component (Fig. 8 b) has previously been considered as a separate and distinct function. It probably represents the range of variation in the amplitude of the "segmental" waves. The range and relative amplitudes of the frequencies constituting this component are found by Fourier transformation. The results are shown in the pressure intensity spectrum in Fig. 8 b. The frequency range is 0 to 25.7 cycles per minute and the pressure intensity range is 0.30 to 8.96 mm$^2$ Hg seconds. The contribution of this component to the total activity is 0.89 mm$^2$ Hg.
Examination of the irradiation record of rabbit 2 (Fig. 6 b) reveals periods of moderate activity interspersed with periods of little or no activity. A wave having a rate of 2.1 cycles per minute is also noted on the autocorrelation function of this record. The length of the record is insufficient to permit adequate evaluation of this low frequency wave.

More thorough analysis of intestinal motility by this technique will require major changes in equipment. The actual process of preparing the autocorrelation functions of these motility records has been most time-consuming, even with the aid of calculating machines to perform the actual computations. The mechanical correlator requires two human operators and many hours are necessary to perform a single autocorrelation. The alternative method of measuring points on the motility record is even more arduous.

Improvements in equipment which permit rapid analysis of longer records may allow more accurate characterization of low frequency components.

On the basis of work now in progress on the application of this method to the analysis of balloon kymographic records of intestinal motor activity, we feel that this method may be used to analyze records detected by different systems. The application of the method to the analysis of the effect of drugs on intestinal motility may make the evaluation of antispasmodics more satisfactory. The analysis of normal human intestinal motility records may yield valuable basic physiological information.

CONCLUSIONS

The application of statistical communication techniques to the analysis of intestinal motility records has made available certain information that has heretofore been obscure. This information is:

1. A measure of total activity expressed in square millimeters of Hg.
2. The frequency and amplitude of segmental activity.
3. A measure of the random variations in the amplitude of the "segmental" waves.
4. A measure of random activity and the range and relative amplitudes of the frequencies making up this component.

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BIBLIOGRAPHY