THE RATE OF GROWTH OF THE DAIRY COW.

V. EXTRAUTERINE GROWTH IN LINEAR DIMENSIONS.

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The curves of growth of animals which have been studied—including that of the dairy cow—show a number of more or less distinct growth cycles or growth epochs. The mathematical analyses of these cycles or epochs have proved very productive. On the other hand, relatively little has been accomplished in the mathematical analysis of the growth curve as a whole. The object of this communication is to briefly present the results of a study of the extrauterine growth of the dairy cow in linear dimensions with the object of formulating an equation or a law representing the general course of growth as contrasted to the equations representing growth cycles or growth epochs.

The data on which this study is based have been accumulating in this department for some time in connection with an investigation concerning the effect of the age of first calving, and the liberality of the food supply on the growth of the dairy cow. The methods of measurements and other details of this investigation, which were formulated and which were carried on for a considerable period of time by Eckles and his students, are discussed in detail in a bulletin of this station. Mr. Swett of this department has also made a

1 Brody, S., and Ragsdale, A. C., J. Gen Physiol., 1920–21, iii, 623.
considerable contribution to these data in connection with an investigation which is being carried on concerning the minimum protein requirements for cattle during growth. For present purposes the details of this investigation may be summarized by stating that twenty-one linear measurements were taken of the cows at monthly intervals during the first 60 months of extrauterine life. The animals under observation consisted of Jersey, Holstein, Ayrshire, and a few milking Shorthorn cattle. Each breed was divided into subgroups according to the age of first calving, and the liberality of the ration they received.

All these data have been prepared for publication in connection with this study. However, the magnitude of these data makes their publication impractical. In this communication, therefore, the results for only one breed, that of Jersey cattle (average of all subgroups), are presented, and these only graphically as shown in Fig. 1. To avoid confusion only eleven of the twenty-one measurements are plotted in this figure. 4

In Fig. 1 are given the observed values and also curves representing the equation

\[ Y = A - Be^{-\frac{t}{9.4}} \tag{1} \]

which was fitted to the data. In this equation \( Y \) represents the values of the measurements at the ages, \( t \) (months). The value 9.4 in the exponent represents the time in months of intrauterine life of this animal, \( e \), the base of natural logarithms, \( A \), a constant representing the values of the measurements at maturity, and \( B \) is another constant. The minus sign in the exponent indicates, of course, that the speed of growth decreases with age. The agreement between observed and computed values is seen to be fairly satisfactory.

The significance of this fairly satisfactory agreement between observed and computed values, consists principally in the fact that this agreement can probably be interpreted on a rational basis. Equation (1) from which the curves were plotted, has the same form

\[ Y = A - Be^{-\frac{t}{9.4}} \]

4 Copies of all the numerical data of this investigation will be deposited in the University of Missouri Library for the benefit of those who may desire to make use of this material for other studies on growth.
Fig. 1. Growth in linear dimensions of Jersey cattle. The smooth curves passing through the observed values represent the equation

$$Y = A - Be^{-k(1 + \pi + \phi)}$$

The numerals refer to the measurements as indicated in Fig. 2, as follows: 18 heart girth, 15 from point of shoulder to ischium, 1 height at withers, 14 from point of shoulder to a point of hips, 12 from highest point of withers to a line between hips, 4 depth of chest just behind elbow joint, 8 length from poll to point of muzzle, 6 width of hips, 10 circumference of muzzle at opening of mouth, 5 width of chest just behind elbow joint, and 9 width of forehead.
Fig. 2. The points of anatomy, and the measurements taken of the cows referred to in Fig. 1.
as the equation of a monomolecular chemical reaction. This naturally suggests the interpretation that the fundamental course of growth is limited by a monomolecular process, and that the cyclic phenomena are subsidiary processes in the fundamentally exponential course of growth.

![Graph showing growth in height at withers plotted to show the relative agreement between observed and computed values using the formulae](image)

Equation (1) differs from the equation of a monomolecular reaction only in the fact that in this equation the numerical values of the constants $A$ and $B$ differ, while in the equation of a monomolecular reaction...
change they are the same. When \( A \) and \( B \) have the same value the curves begin at zero; if \( A \) and \( B \) differ, the theoretical curve does not begin at zero. As seen in Fig. 1, the computed values do not begin at conception, which is presumably the beginning of growth in linear dimensions, but some time between conception and birth. It is, of course, possible that the development of certain structures which materially affect the linear values begin some time after conception.

![Graph showing growth in weight of Jersey cattle](image)

**Fig. 4. Growth in weight of Jersey cattle.** Plotted to show the agreement between observed values as given in a preceding communication,¹ and values computed from the exponential equation

\[
Y = 950 - 925e^{-0.06t}.
\]

The rise between 20 and 29 months is due to gestation, and the sudden decline at about 29 months is due to calving. The animals were bred at about 20 months of age and calved at about 29 months of age.

as shown by the curves. This possibility cannot, of course, be present as regards growth in weight (Fig. 4). It may also be that the beginning of the growth process is irregular in the sense that the course of the process at the beginning of growth is somewhat different from that of the course of the process after it is well started.
Such irregularities at the beginning of a process are of frequent occurrence in chemistry. The relation between the curves of height at withers as plotted from equation (1) in which the numerical values of $A$ and $B$ differ, and the same equation in which the numerical values of $A$ and $B$ are the same, are shown in Fig. 3. From Fig. 1 it is clear that the extrauterine period of growth certainly follows an exponential course. From Fig. 3 it appears probable that the course of growth in linear dimensions is not only exponential, but that it also follows the course of monomolecular change.

Fluctuations due to cyclic phenomena are, of course, more pronounced in the curves of growth in weight than in the curves of growth in linear dimensions, inasmuch as weight tends to increase as the cube of increase in linear dimensions. However, the general tendency of the curve of extrauterine growth in weight appears to follow an exponential course as shown by the fair agreement between observed and computed values in Fig. 4.

**SUMMARY.**

Barring fluctuations due to the cyclic phenomena, the extrauterine course of growth in linear dimensions and in weight of the dairy cow follows an exponential law having the same form as the law representing the course of monomolecular change in chemistry. This suggests the interpretation that the general course of growth is limited by a monomolecular chemical process, and that the cyclic phenomena are due to subsidiary processes in the fundamentally exponential course of growth.

The fact that growth follows or tends to follow an exponential course may be stated more simply as follows: if the unit of time is taken sufficiently large so that fluctuations due to the cyclic phenomena are balanced or eliminated, then the amount of growth made during the given unit of time at any age tends to be a constant percentage of the growth made during the preceding unit of time. Thus, the growth in height at withers made during any year is about 34 per cent of the growth made during the preceding year. Similarly the growth in weight made during any year is about 56 per cent of the growth in weight made during the preceding year. This is in
accordance with expectations if it is assumed that each animal begins life with a definite endowment of limiting substance necessary for the process of growth, and that this endowment is used up at a constant rate (or percentage) of itself.