A NOTE ON THE SIMILARITIES BETWEEN THE CURVES OF GROWTH AND OF REGENERATION.

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(Received for publication, March 24, 1924.)

In a preceding communication on the growth of the dairy cow, it was shown that the course of extrauterine growth of this animal can be satisfactorily represented by the equation

\[ Y = A - Be^{-kt} \]  

(1)

in which \( Y \) is the weight or any linear measure at the age, \( t \), \( e \) the base of natural logarithms, \( k \) the velocity constant of growth, and \( A \) and \( B \) are constants, \( A \) having a smaller numerical value than \( B \). Since there is such a difference between the values of \( A \) and \( B \), the curve of equation (1) when extrapolated to include the intrauterine period of growth does not cut the time axis at zero. Instead of beginning at conception, it begins sometimes after, as illustrated in Fig. 1a, thus indicating either a relatively slow phase of growth of the fetus in the early stages or a vigorous growth, but of a type which cannot be expressed in terms of weight or linear measurements. It may also be, of course, that an error was made in the choice of the equation to represent the course of growth, that is the equation (1) may be a purely empirical one representing only a part of the course of growth. Recently the writer chanced to find a paper which promised to throw some light on this problem. Miss Durbin, studying the rate of regeneration of the tail of the tadpole \( Rana clamitans \), found an apparently slow phase of regeneration during the first 3 to 5 days following the removal of part of the tail, and she advanced a very plausible explanation for this apparent slow phase of regeneration.

This suggested the possibility that there may be a similar slow phase, and for the same reasons, in the beginning of growth of the fetus, and it was, therefore, thought interesting to compare the course of growth and of regeneration in order to bring out such a similarity if it exists.

Accordingly, equation (1) was fitted to the data on regeneration of the tail of the tadpole with the results shown in Fig. 1, b. A comparison between Figs. 1a and 1b shows that the agreements and the discrepancies between observed and computed values are of the same order for the curves of growth and regeneration. It appears then, that the course of growth and the course of regeneration are governed by the same laws and that the apparent slow initial phases of growth and of regeneration are due to the same causes. Miss Durbin observed the slow initial phase of regeneration and on the basis of histological studies of the regenerating tissues she explained it as due, in part at least, to the formation of a cap of embryonic cells which serves as a basis for the more active regeneration. In other words, regeneration is vigorous from the beginning, but it is principally qualitative and cannot, therefore, be fully expressed in terms of linear measurements.

If growth and regeneration are governed by the same laws as was assumed in the last paragraph, then it is natural to extend the explanation for the slow phase of regeneration to include the slow phase of growth. That is, that growth in the early stages like regeneration in the early stages is vigorous, but the growth is principally qualitative and cannot be fully represented in terms of grams or centimeters; and that if these qualitative changes at the beginning of growth or of regeneration could be converted into some common unit of growth energy with the subsequent measurable quantitative changes, the apparent initial lag (a on Fig. 1) would disappear, and with this the differences in the numerical values of A and B in equation (1) would disappear. This comparison between the curves of growth and of regeneration, therefore, leads to the tentative conclusion that the initial lag in the curves is only apparent due to our inability to convert the initial qualitative growth into quantitative units, and that equation (1) is rational potentially capable of expressing the whole course of growth and of regeneration.

Incidentally, the comparison between the curves of growth and of regeneration made in this note adds the following to our knowledge.
Fig. 1. The relative course of growth and of regeneration. The circles and crosses represent the observed values, the smooth curves represent equation (1) given in the text.
of the phenomenon of regeneration. First, equation (1) is presented which represents the course of regeneration. An unsatisfactory verbal description of the course of regeneration is thus replaced by a mathematical expression stating that the ratio of the amount of regeneration during a given unit of time to the amount of regeneration during the preceding unit of time tends to be the same for all stages of the process of regeneration—at any rate after the first few days after the operation. Moreover, as pointed out, this equation appears to be rational, and it may, therefore, be tentatively considered as expressing a law of regeneration. Second, this equation is the same as that used to represent the course of growth of an individual. This may be taken to substantiate the theory that regeneration and growth are essentially identical processes governed by the same laws. Third, this equation may be considered to be the same as that used to represent the course of accumulation of the product of a monomolecular chemical reaction, or of a series of chemical reactions limited by a monomolecular reaction. This may be taken to substantiate the theory that regeneration as well as growth, is limited by a physicochemical process.

SUMMARY.

The curves of growth and of regeneration follow the same course, and can be represented by the same exponential equation. This is taken to substantiate the theory that growth and regeneration are essentially identical processes governed by the same laws.

A common peculiarity of the curves of growth and of regeneration is that during a short period in the early stages of regeneration and of growth, the apparent observed speed of these processes seems to be relatively slow. As a result, the curve of the fitted equation cuts the time axis not at zero, the beginning of growth or regeneration, but somewhat later. Data on regeneration are cited indicating that the initial slow phase of regeneration is due to the time required for the formation of a cap of embryonic cells which serves as a basis for the more active later regeneration; in other words, to qualitative growth which cannot be expressed in terms of quantitative units. It is suggested that the apparent initial slow phase of growth of the individual from the fertilized egg is due to a similar qualitative growth. It is suggested that if the initial qualitative changes could be con-
verted into some common unit with the subsequent quantitative changes, the apparent initial lag would disappear, and the exponential equation representing the course of these processes would then be the same as the equation used to represent the course of a monomolecular chemical reaction.

Certain implications of this reasoning are discussed in the text.